EXERCISE- 5.1

Question 1:

Express the given complex number in the form $a + ib$: $(5i)\left(-\frac{3}{5}i\right)$

$$
\begin{aligned}
(5i)\left(\frac{-3}{5}i\right) &= -5\times\frac{3}{5}\times i\times i \\
&= -3i^2 \\
&= -3(-1) \qquad \left[i^2=-1\right] \\
&= 3
\end{aligned}
$$

Question 2:

Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

$$
\begin{aligned}
i^9+i^{19} &= i^{4\times2+1}+i^{4\times4+3} \\
&= \left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3 \\
&= 1\times i+1\times(-i) \qquad \left[i^4=1,\ i^3=-i\right] \\
&= i+(-i) \\
&= 0
\end{aligned}
$$

Question 3:

Express the given complex number in the form $a + ib$: i^{-39}

$$
\begin{aligned}
i^{-39} &= i^{-4\times9-3}=\left(i^4\right)^{-9}\cdot i^{-3} \\
&= (1)^{-9}\cdot i^{-3} \qquad \left[i^4=1\right] \\
&= \frac{1}{i^3}=\frac{1}{-i} \qquad \left[i^3=-i\right] \\
&= \frac{-1}{i}\times\frac{i}{i} \\
&= \frac{-i}{i^2}=\frac{-i}{-1}=i \qquad \left[i^2=-1\right]
\end{aligned}
$$

Question 4:

Express the given complex number in the form $a + ib$: $3(7 + i7) + i(7 + i7)$

$$\begin{aligned}3(7+i7)+i(7+i7) &= 21+21i+7i+7i^2 \\ &= 21+28i+7\times(-1) \qquad [\because i^2=-1] \\ &= 14+28i\end{aligned}$$

Question 5:

Express the given complex number in the form $a + ib$: $(1 - i) - (-1 + i6)$

$$\begin{aligned}(1-i)-(-1+i6) &= 1-i+1-6i \\ &= 2-7i\end{aligned}$$

Question 6:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

$$\begin{aligned}&\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right) \\ &=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i \\ &=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right) \\ &=\frac{-19}{5}+i\left(\frac{-21}{10}\right) \\ &=\frac{-19}{5}-\frac{21}{10}i\end{aligned}$$

Question 7:

Express the given complex number in the form $a + ib$: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Question 8:

Express the given complex number in the form $a + ib$: $(1 - i)^4$

$$(1-i)^4=\left[(1-i)^2\right]^2$$
$$=\left[1^2+i^2-2i\right]^2$$
$$=\left[1-1-2i\right]^2$$
$$=(-2i)^2$$
$$=(-2i)\times(-2i)$$
$$=4i^2=-4 \qquad \left[i^2=-1\right]$$

Question 9:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3}+3i\right)^3$

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]$$
$$=\frac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]$$
$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$
$$=\frac{-242}{27}-26i$$

Question 10:

Express the given complex number in the form *a* + *ib*: $\left(-2-\frac{1}{3}i\right)^3$

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number $4 - 3i$

Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2+(-3)^2 = 16+9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Let $z = \sqrt{5}+3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number $-i$

Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

Question 14:

Express the following expression in the form of $a + ib$.

$$\frac{\left(3 + i\sqrt{5}\right)\left(3 - i\sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{2}i\right) - \left(\sqrt{3} - i\sqrt{2}\right)}$$

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

EXERCISE- 5.2

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+\left(-\sqrt{3}\right)^2$$

$$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+3$$

$$\Rightarrow r^2=4 \qquad \left[\cos^2\theta+\sin^2\theta=1\right]$$

$$\Rightarrow r=\sqrt{4}=2 \qquad \left[\text{Conventionally, } r>0\right]$$

$$\therefore \text{Modulus}=2$$

$$\therefore\ 2\cos\theta=-1 \text{ and } 2\sin\theta=-\sqrt{3}$$

$$\Rightarrow \cos\theta=\frac{-1}{2} \text{ and } \sin\theta=\frac{-\sqrt{3}}{2}$$

Since both the values of sin θ and cos θ are negative and sinθ and cosθ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow r^2 = 3+1 = 4 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$$
$$\Rightarrow r = \sqrt{4} = 2 \qquad \left[\text{Conventionally, } r > 0\right]$$
$$\therefore \text{Modulus } = 2$$
$$\therefore\ 2\cos\theta = -\sqrt{3} \text{ and } 2\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \qquad \left[\text{As } \theta \text{ lies in the II quadrant}\right]$$

Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: $1 - i$

$1 - i$

Let $r \cos \theta = 1$ and $r \sin \theta = -1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2$$
$$\Rightarrow r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1$$
$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

$$\therefore 1 - i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos\left(-\frac{\pi}{4}\right) + i\sqrt{2} \sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Question 4:

Convert the given complex number in polar form: $-1 + i$

$-1 + i$

Let $r \cos \theta = -1$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1+i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: – 1 – i

– 1 – i

Let r cos θ = –1 and r sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

$\therefore -1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4} = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$ This is the required polar form.

Question 6:

Convert the given complex number in polar form: –3

–3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 9$$
$$\Rightarrow r^2 = 9$$
$$\Rightarrow r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$
$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$
$$\therefore \theta = \pi$$

$$\therefore -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3\left(\cos\pi + i\sin\pi\right)$$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3}+i$

$\sqrt{3}+i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3 + 1$$
$$\Rightarrow r^2 = 4$$
$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$$

$$\therefore \sqrt{3} + i = r\cos\theta + i\,r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: *i*

i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$
$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

EXERCISE- 5.3

Question 1:

Solve the equation $x^2 + 3 = 0$

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 0$, and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2 \times 1} = \frac{\pm\sqrt{12}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{\pm 2\sqrt{3}\,i}{2} = \pm\sqrt{3}\,i$$

Question 2:

Solve the equation $2x^2 + x + 1 = 0$

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = 1$, and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm \sqrt{7}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$

Question 3:

Solve the equation $x^2 + 3x + 9 = 0$

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 3$, and $c = 9$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 4:

Solve the equation $-x^2 + x - 2 = 0$

The given quadratic equation is $-x^2 + x - 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = -1$, $b = 1$, and $c = -2$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7}\, i}{-2} \qquad \left[\sqrt{-1} = i\right]$$

Question 5:

Solve the equation $x^2 + 3x + 5 = 0$

The given quadratic equation is $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 3$, and $c = 5$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 6:

Solve the equation $x^2 - x + 2 = 0$

The given quadratic equation is $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = -1$, and $c = 2$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Question 7:

Solve the equation $\sqrt{2}x^2 + x + \sqrt{2} = 0$

The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

Question 8:

Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3}$, $b = -\sqrt{2}$, and $c = 3\sqrt{3}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = \left(-\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(3\sqrt{3}\right) = 2 - 36 = -34$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}\, i}{2\sqrt{3}} \qquad \left[\sqrt{-1} = i\right]$$

Question 9:

Solve the equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$

The given quadratic equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

This equation can also be written as $\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$

$\therefore$ Discriminant $(D) = b^2 - 4ac = \left(\sqrt{2}\right)^2 - 4 \times \left(\sqrt{2}\right) \times 1 = 2 - 4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{2} \pm \sqrt{2 - 4\sqrt{2}}}{2 \times \sqrt{2}} = \frac{-\sqrt{2} \pm \sqrt{2\left(1 - 2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$= \left(\frac{-\sqrt{2} \pm \sqrt{2}\left(\sqrt{2\sqrt{2} - 1}\right) i}{2\sqrt{2}}\right) \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{-1 \pm \left(\sqrt{2\sqrt{2} - 1}\right) i}{2}$$

Question 10:

Solve the equation $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

The given quadratic equation is $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

This equation can also be written as $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

$\therefore$ Discriminant $(\mathrm{D}) = b^2 - 4ac = 1^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{\mathrm{D}}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$