EXERCISE:- 11.1

Question 1:

Find the equation of the circle with centre (0, 2) and radius 2

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (0, 2)$ and radius $(r) = 2$.

Therefore, the equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

Question 2:

Find the equation of the circle with centre (–2, 3) and radius 4

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (-2, 3)$ and radius $(r) = 4$.

Therefore, the equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r) = \frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$
$$144x^2-144x+36+144y^2-72y+9-1=0$$
$$144x^2-144x+144y^2-72y+44=0$$
$$36x^2-36x+36y^2-18y+11=0$$
$$36x^2+36y^2-36x-18y+11=0$$

Question 4:

Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$$
$$x^2-2x+1+y^2-2y+1=2$$
$$x^2+y^2-2x-2y=0$$

Question 5:

Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$

The equation of a circle with centre (h, k) and radius r is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre $(h, k) = (-a, -b)$ and radius $(r) = \sqrt{a^2 - b^2}$.

Therefore, the equation of the circle is

$$(x+a)^2 + (y+b)^2 = \left(\sqrt{a^2 - b^2}\right)^2$$
$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$
$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

Question 6:

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$(x + 5)^2 + (y - 3)^2 = 36$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is $(-5, 3)$, while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$x^2 + y^2 - 4x - 8y - 45 = 0$

$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$

$\Rightarrow \{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4) + 4^2\} - 4 - 16 = 45$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 65$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = \left(\sqrt{65}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$, and $r = \sqrt{65}$.

Thus, the centre of the given circle is (2, 4), while its radius is $\sqrt{65}$.

Question 8:

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$x^2 + y^2 - 8x + 10y - 12 = 0$

$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$

$\Rightarrow \{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$

$\Rightarrow (x-4)^2 + \{y-(-5)\}^2 = \left(\sqrt{53}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is (4, –5), while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$2x^2 + 2y^2 - x = 0$

$\Rightarrow \left(2x^2 - x\right) + 2y^2 = 0$

$\Rightarrow 2\left[\left(x^2 - \frac{x}{2}\right) + y^2\right] = 0$

$\Rightarrow \left\{x^2 - 2.x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2\right\} + y^2 - \left(\frac{1}{4}\right)^2 = 0$

$\Rightarrow \left(x - \frac{1}{4}\right)^2 + \left(y - 0\right)^2 = \left(\frac{1}{4}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$, and $r = \frac{1}{4}$.

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0\right)$, while its radius is $\frac{1}{4}$.

Question 10:

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$(4 - h)^2 + (1 - k)^2 = r^2$ … (1)

$(6 - h)^2 + (5 - k)^2 = r^2$ … (2)

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$4h + k = 16$ … (3)

From equations (1) and (2), we obtain

$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$

$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$

$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$

$\Rightarrow 4h + 8k = 44$

$\Rightarrow h + 2k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$(4 - 3)^2 + (1 - 4)^2 = r^2$

$\Rightarrow (1)^2 + (-3)^2 = r^2$

$\Rightarrow 1 + 9 = r^2$

$\Rightarrow r^2 = 10$

$\Rightarrow r = \sqrt{10}$

Thus, the equation of the required circle is

$(x - 3)^2 + (y - 4)^2 = \left(\sqrt{10}\right)^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$

Question 11:

Find the equation of the circle passing through the points (2, 3) and (–1, 1) and whose centre is on the line $x - 3y - 11 = 0$.

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (2, 3) and (–1, 1),

$(2 - h)^2 + (3 - k)^2 = r^2$ … (1)

$(-1 - h)^2 + (1 - k)^2 = r^2$ … (2)

Since the centre (h, k) of the circle lies on line $x - 3y - 11 = 0$,

$h - 3k = 11$ … (3)

From equations (1) and (2), we obtain

$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$

$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$

$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$

$\Rightarrow 6h + 4k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2+\left(\frac{2y+5}{2}\right)^2=\frac{130}{4}$$

$$4x^2-28x+49+4y^2+20y+25=130$$

$$4x^2+4y^2-28x+20y-56=0$$

$$4(x^2+y^2-7x+5y-14)=0$$

$$x^2+y^2-7x+5y-14=0$$

Question 12:

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x-h)^2+y^2=25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2-h)^2+3^2=25$$

$$\Rightarrow (2-h)^2=25-9$$

$$\Rightarrow (2-h)^2=16$$

$$\Rightarrow 2-h=\pm\sqrt{16}=\pm 4$$

If $2-h=4$, then $h=-2$.

If $2-h=-4$, then $h=6$.

When $h = -2$, the equation of the circle becomes

$(x+2)^2+y^2=25$

$x^2+4x+4+y^2=25$

$x^2+y^2+4x-21=0$

When $h = 6$, the equation of the circle becomes

$(x-6)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 + y^2 - 12x + 11 = 0$

Question 13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Let the equation of the required circle be $(x-h)^2 + (y-k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$(0-h)^2 + (0-k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$

The equation of the circle now becomes $(x-h)^2 + (y-k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$(a-h)^2 + (0-k)^2 = h^2 + k^2 \dots (1)$

$(0-h)^2 + (b-k)^2 = h^2 + k^2 \dots (2)$

From equation (1), we obtain

$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$\Rightarrow a^2 - 2ah = 0$

$\Rightarrow a(a-2h) = 0$

$\Rightarrow a = 0$ or $(a-2h) = 0$

However, $a \neq 0$; hence, $(a-2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain

$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$\Rightarrow b^2 - 2bk = 0$

$\Rightarrow b(b - 2k) = 0$

$\Rightarrow b = 0$ or $(b - 2k) = 0$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \frac{b}{2}$.

Thus, the equation of the required circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$
$$\Rightarrow\left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$
$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$
$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$
$$\Rightarrow x^2+y^2-ax-by=0$$

Question 14:

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$
$$x^2-4x+4+y^2-4y+4=13$$
$$x^2+y^2-4x-4y-5=0$$

Question 15:

Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 0$, and$r = 5$.

$\therefore$Centre = (0, 0) and radius = 5

Distance between point (–2.5, 3.5) and centre (0, 0)

$$=\sqrt{(-2.5-0)^2+(3.5-0)^2}$$
$$=\sqrt{6.25+12.25}$$
$$=\sqrt{18.5}$$
$$=4.3\ (\text{approx.})<5$$

Since the distance between point (–2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (–2.5, 3.5) lies inside the circle.

EXERCISE:- 11.2

Question 1:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$

The given equation is $y^2 = 12x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 12 \Rightarrow a = 3$

$\therefore$Coordinates of the focus = $(a, 0) = (3, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of direcctrix, $x = -a$ i.e., $x = -3$ i.e., $x + 3 = 0$

Length of latus rectum = $4a = 4 \times 3 = 12$

Question 2:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$

The given equation is $x^2 = 6y$.

Here, the coefficient of y is positive. Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

$\therefore$Coordinates of the focus = $(0, a)$ = $\left(0, \frac{3}{2}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$ i.e., $y = -\frac{3}{2}$

Length of latus rectum = $4a = 6$

Question 3:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = -8x$

The given equation is $y^2 = -8x$.

Here, the coefficient of x is negative. Hence, the parabola opens towards the left.

On comparing this equation with $y^2 = -4ax$, we obtain

$-4a = -8 \Rightarrow a = 2$

$\therefore$Coordinates of the focus = $(-a, 0) = (-2, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = a$ i.e., $x = 2$

Length of latus rectum = $4a = 8$

Question 4:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -16y$

The given equation is $x^2 = -16y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$-4a = -16 \Rightarrow a = 4$

$\therefore$Coordinates of the focus = $(0, -a) = (0, -4)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$ i.e., $y = 4$

Length of latus rectum = $4a = 16$

Question 5:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$

The given equation is $y^2 = 10x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

∴Coordinates of the focus = $(a, 0)$ $= \left(\frac{5}{2}, 0\right)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = -a, \text{ i.e., } x = -\frac{5}{2}$

Length of latus rectum = $4a = 10$

Question 6:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -9y$

The given equation is $x^2 = -9y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$$-4a = -9 \Rightarrow b = \frac{9}{4}$$

∴Coordinates of the focus = $(0, -a) = \left(0, -\frac{9}{4}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$, i.e., $y = \frac{9}{4}$

Length of latus rectum = $4a = 9$

Question 7:

Find the equation of the parabola that satisfies the following conditions: Focus (6, 0); directrix $x = –6$

Focus (6, 0); directrix, $x = –6$

Since the focus lies on the x-axis, the x-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y^2 = 4ax$ or

$y^2 = – 4ax$.

It is also seen that the directrix, $x = –6$ is to the left of the y-axis, while the focus (6, 0) is to the right of the y-axis. Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

Thus, the equation of the parabola is $y^2 = 24x$.

Question 8:

Find the equation of the parabola that satisfies the following conditions: Focus (0, –3); directrix $y= 3$

Focus = (0, –3); directrix $y = 3$

Since the focus lies on the y-axis, the y-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2 = 4ay$ or

$x^2 = – 4ay$.

It is also seen that the directrix, $y = 3$ is above the x-axis, while the focus

(0, –3) is below the x-axis. Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

Thus, the equation of the parabola is $x^2 = -12y$.

Question 9:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0); focus (3, 0)

Vertex (0, 0); focus (3, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = 4ax$.

Since the focus is (3, 0), $a = 3$.

Thus, the equation of the parabola is $y^2 = 4 \times 3 \times x$, i.e., $y^2 = 12x$

Question 10:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) focus (–2, 0)

Vertex (0, 0) focus (–2, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$.

Since the focus is (–2, 0), $a = 2$.

Thus, the equation of the parabola is $y^2 = -4(2)x$, i.e., $y^2 = -8x$

Question 11:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) passing through (2, 3) and axis is along x-axis

Since the vertex is (0, 0) and the axis of the parabola is the x-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

The parabola passes through point (2, 3), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point

(2, 3) must satisfy the equation $y^2 = 4ax$.

$$\therefore 3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$
$$y^2 = \frac{9}{2}x$$
$$2y^2 = 9x$$

Question 12:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis

Since the vertex is (0, 0) and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point

(5, 2) must satisfy the equation $x^2 = 4ay$.

$$\therefore (5)^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$x^2 = 4\left(\frac{25}{8}\right)y$

$2x^2 = 25y$

EXERCISE:- 11.3

Question 1:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 6$ and $b = 4$.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36-16} = \sqrt{20} = 2\sqrt{5}$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = $2a = 12$

Length of minor axis = $2b = 8$

Eccentricity, $e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times16}{6} = \frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 2$ and $a = 5$.

$\therefore c = \sqrt{a^2-b^2} = \sqrt{25-4} = \sqrt{21}$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = $2a = 10$

Length of minor axis = $2b = 4$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times4}{5} = \frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 4$ and $b = 3$.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $(\pm\sqrt{7},0)$.

The coordinates of the vertices are $(\pm 4,0)$.

Length of major axis = $2a = 8$

Length of minor axis = $2b = 6$

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 5$ and $a = 10$.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = $2a$ = 20

Length of minor axis = $2b$ = 10

Eccentricity, $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

The given equation is $\frac{x^2}{49}+\frac{y^2}{36}=1$ or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 7$ and $b = 6$.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13},0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = $2a = 14$

Length of minor axis = $2b = 12$

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 10$ and $a = 20$.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$.

The coordinates of the vertices are $(0, \pm 20)$

Length of major axis = $2a = 40$

Length of minor axis = $2b = 20$

Eccentricity, $e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 100}{20}=10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$
$$\text{Or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$
$$\text{Or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 2$ and $a = 6$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$.

The coordinates of the vertices are $(0, \pm 6)$.

Length of major axis $= 2a = 12$

Length of minor axis $= 2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$
$$\text{Or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$
$$\text{Or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are $(0, \pm 4)$.

Length of major axis = $2a = 8$

Length of minor axis = $2b = 2$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2 + 9y^2 = 36$$
$$\text{Or, } \frac{x^2}{9} + \frac{y^2}{4} = 1$$
$$\text{Or, } \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 3$ and $b = 2$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5}, 0\right)$.

The coordinates of the vertices are $(\pm 3, 0)$.

Length of major axis $= 2a = 6$

Length of minor axis $= 2b = 4$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±5, 0), foci (±4, 0)

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is the semi-major axis.

Accordingly, $a = 5$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 5^2 = b^2 + 4^2$$
$$\Rightarrow 25 = b^2 + 16$$
$$\Rightarrow b^2 = 25 - 16$$
$$\Rightarrow b = \sqrt{9} = 3$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, $a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 13^2 = b^2 + 5^2$$
$$\Rightarrow 169 = b^2 + 25$$
$$\Rightarrow b^2 = 169 - 25$$
$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{12^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{144}+\frac{y^2}{169}=1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $a = 6$, $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$\therefore 6^2 = b^2 + 4^2$

$\Rightarrow 36 = b^2 + 16$

$\Rightarrow b^2 = 36 - 16$

$\Rightarrow b = \sqrt{20}$

Thus, the equation of the ellipse is $\dfrac{x^2}{6^2} + \dfrac{y^2}{\left(\sqrt{20}\right)^2} = 1$ or $\dfrac{x^2}{36} + \dfrac{y^2}{20} = 1$.

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where *a* is the semi-major axis.

Accordingly, $a = 3$ and $b = 2$.

Thus, the equation of the ellipse is $\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} = 1$ i.e., $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$.

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0, \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Ends of major axis $\left(0, \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$, where *a* is the semi-major axis.

Accordingly, $a = \sqrt{5}$ and $b = 1$.

Thus, the equation of the ellipse is $\frac{x^2}{1^2} + \frac{y^2}{\left(\sqrt{5}\right)^2} = 1$ or $\frac{x^2}{1} + \frac{y^2}{5} = 1$.

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci (±5, 0)

Length of major axis = 26; foci = (±5, 0).

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Accordingly, $2a = 26 \Rightarrow a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 13^2 = b^2 + 5^2$$
$$\Rightarrow 169 = b^2 + 25$$
$$\Rightarrow b^2 = 169 - 25$$
$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$ or $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci (0, ±6)

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, $2b = 16 \Rightarrow b = 8$ and $c = 6$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$$
$$\Rightarrow a = \sqrt{100} = 10$$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), $a = 4$

Foci (±3, 0), $a = 4$

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $c = 3$ and $a = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 4^2 = b^2 + 3^2$$
$$\Rightarrow 16 = b^2 + 9$$
$$\Rightarrow b^2 = 16 - 9 = 7$$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

Question 18:

Find the equation for the ellipse that satisfies the given conditions: $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

It is given that $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $b = 3$, $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$\Rightarrow a = 5$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \quad ...(2)$$

$$\frac{1}{b^2}+\frac{36}{a^2}=1 \quad ...(3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$ or $4x^2+y^2=40$.

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Since the major axis is on the x-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \quad ...(2)$$

$$\frac{36}{a^2}+\frac{4}{b^2}=1 \quad ...(3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$ or $x^2+4y^2=52$.

EXERCISE:- 11.4

Question 1:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$.

The given equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}-\frac{y^2}{3^2}=1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we obtain a= 4 and b = 3.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 4^2 + 3^2 = 25$$
$$\Rightarrow c = 5$$

Therefore,

The coordinates of the foci are (±5, 0).

The coordinates of the vertices are (±4, 0).

Eccentricity, $e = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

Question 2:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{y^2}{9}-\frac{x^2}{27}=1$

The given equation is $\frac{y^2}{9}-\frac{x^2}{27}=1$ or $\frac{y^2}{3^2}-\frac{x^2}{\left(\sqrt{27}\right)^2}=1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain a= 3 and $b=\sqrt{27}$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 3^2 + \left(\sqrt{27}\right)^2 = 9 + 27 = 36$$
$$\Rightarrow c = 6$$

Therefore,

The coordinates of the foci are (0, ±6).

The coordinates of the vertices are (0, ±3).

Eccentricity, $e = \frac{c}{a} = \frac{6}{3} = 2$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 27}{3} = 18$

Question 3:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $9y^2 - 4x^2 = 36$

The given equation is $9y^2 - 4x^2 = 36$.

It can be written as

$9y^2 - 4x^2 = 36$

$$\text{Or, } \frac{y^2}{4} - \frac{x^2}{9} = 1$$
$$\text{Or, } \frac{y^2}{2^2} - \frac{x^2}{3^2} = 1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain a= 2 and b = 3.

We know that $a^2 + b^2 = c^2$.

$\therefore c^2 = 4 + 9 = 13$
$\Rightarrow c = \sqrt{13}$

Therefore,

The coordinates of the foci are $\left(0,\ \pm\sqrt{13}\right)$.

The coordinates of the vertices are $\left(0,\ \pm 2\right)$.

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{2} = 9$

Question 4:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$

The given equation is $16x^2 - 9y^2 = 576$.

It can be written as

$16x^2 - 9y^2 = 576$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$$
$$\Rightarrow \frac{x^2}{6^2} - \frac{y^2}{8^2} = 1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain $a = 6$ and $b = 8$.

We know that $a^2 + b^2 = c^2$.

$\therefore c^2 = 36 + 64 = 100$
$\Rightarrow c = 10$

Therefore,

The coordinates of the foci are (±10, 0).

The coordinates of the vertices are (±6, 0).

Eccentricity, $e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 64}{6} = \frac{64}{3}$

Question 5:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $5y^2 - 9x^2 = 36$

The given equation is $5y^2 - 9x^2 = 36$.

$$\Rightarrow \frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{4} = 1$$

$$\Rightarrow \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{2^2} = 1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = \frac{6}{\sqrt{5}}$ and $b = 2$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = \frac{36}{5} + 4 = \frac{56}{5}$$

$$\Rightarrow c = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$$

Therefore, the coordinates of the foci are $\left(0,\ \pm\frac{2\sqrt{14}}{\sqrt{5}}\right)$.

The coordinates of the vertices are $\left(0,\ \pm\frac{6}{\sqrt{5}}\right)$.

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{\left(\frac{2\sqrt{14}}{\sqrt{5}}\right)}{\left(\frac{6}{\sqrt{5}}\right)}=\frac{\sqrt{14}}{3}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{\left(\frac{6}{\sqrt{5}}\right)}=\frac{4\sqrt{5}}{3}$

Question 6:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$

The given equation is $49y^2 - 16x^2 = 784$.

It can be written as
$49y^2 - 16x^2 = 784$

$$\text{Or, } \frac{y^2}{16}-\frac{x^2}{49}=1$$
$$\text{Or, } \frac{y^2}{4^2}-\frac{x^2}{7^2}=1 \qquad ...(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain $a= 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 16+49=65$$
$$\Rightarrow c=\sqrt{65}$$

Therefore,

The coordinates of the foci are $\left(0,\ \pm\sqrt{65}\right)$.

The coordinates of the vertices are (0, ±4).

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$

Question 7:

Find the equation of the hyperbola satisfying the give conditions: Vertices (±2, 0), foci (±3, 0)

Vertices (±2, 0), foci (±3, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are (±2, 0), $a = 2$.

Since the foci are (±3, 0), $c = 3$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 2^2 + b^2 = 3^2$$
$$b^2 = 9 - 4 = 5$$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Question 8:

Find the equation of the hyperbola satisfying the give conditions: Vertices (0, ±5), foci (0, ±8)

Vertices (0, ±5), foci (0, ±8)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the vertices are (0, ±5), $a = 5$.

Since the foci are (0, ±8), $c = 8$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 5^2 + b^2 = 8^2$$
$$b^2 = 64 - 25 = 39$$

Thus, the equation of the hyperbola is $\frac{y^2}{25}-\frac{x^2}{39}=1$.

Question 9:

Find the equation of the hyperbola satisfying the give conditions: Vertices (0, ±3), foci (0, ±5)

Vertices (0, ±3), foci (0, ±5)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the vertices are (0, ±3), $a = 3$.

Since the foci are (0, ±5), $c = 5$.

We know that $a^2 + b^2 = c^2$.

$\therefore 3^2 + b^2 = 5^2$

$\Rightarrow b^2 = 25 - 9 = 16$

Thus, the equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$.

Question 10:

Find the equation of the hyperbola satisfying the give conditions: Foci (±5, 0), the transverse axis is of length 8.

Foci (±5, 0), the transverse axis is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the foci are (±5, 0), $c = 5$.

Since the length of the transverse axis is 8, $2a = 8 \Rightarrow a = 4$.

We know that $a^2 + b^2 = c^2$.

$\therefore 4^2 + b^2 = 5^2$

$\Rightarrow b^2 = 25 - 16 = 9$

Thus, the equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Question 11:

Find the equation of the hyperbola satisfying the give conditions: Foci (0, ±13), the conjugate axis is of length 24.

Foci (0, ±13), the conjugate axis is of length 24.

Here, the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the foci are (0, ±13), c = 13.

Since the length of the conjugate axis is 24, $2b = 24 \Rightarrow b = 12$.

We know that $a^2 + b^2 = c^2$.

$\therefore a^2 + 12^2 = 13^2$

$\Rightarrow a^2 = 169 - 144 = 25$

Thus, the equation of the hyperbola is $\frac{y^2}{25}-\frac{x^2}{144}=1$.

Question 12:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are $\left(\pm 3\sqrt{5},\ 0\right)$, $c = \pm 3\sqrt{5}$.

Length of latus rectum = 8

$$\Rightarrow \frac{2b^2}{a}=8$$
$$\Rightarrow b^2 = 4a$$

We know that $a^2 + b^2 = c^2$.

$\therefore a^2 + 4a = 45$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow a^2 + 9a - 5a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = -9, 5$

Since a is non-negative, $a = 5$.

$\therefore b^2 = 4a = 4 \times 5 = 20$

Thus, the equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$.

Question 13:

Find the equation of the hyperbola satisfying the give conditions: Foci (±4, 0), the latus rectum is of length 12

Foci (±4, 0), the latus rectum is of length 12.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the foci are (±4, 0), $c = 4$.

Length of latus rectum = 12

$$\Rightarrow \frac{2b^2}{a} = 12$$
$$\Rightarrow b^2 = 6a$$

We know that $a^2 + b^2 = c^2$.

$\therefore a^2 + 6a = 16$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow a^2 + 8a - 2a - 16 = 0$

$\Rightarrow (a + 8)(a - 2) = 0$

$\Rightarrow a = -8, 2$

Since a is non-negative, $a = 2$.

$\therefore b^2 = 6a = 6 \times 2 = 12$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$.

Question 14:

Find the equation of the hyperbola satisfying the give conditions: Vertices $(\pm 7, 0)$, $e = \frac{4}{3}$

Vertices $(\pm 7, 0)$, $e = \frac{4}{3}$

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are $(\pm 7, 0)$, $a = 7$.

It is given that $e = \frac{4}{3}$

$\therefore \frac{c}{a} = \frac{4}{3}$ $\left[e = \frac{c}{a}\right]$

$\Rightarrow \frac{c}{7} = \frac{4}{3}$

$\Rightarrow c = \frac{28}{3}$

We know that $a^2 + b^2 = c^2$.

$\therefore 7^2 + b^2 = \left(\frac{28}{3}\right)^2$

$\Rightarrow b^2 = \frac{784}{9} - 49$

$\Rightarrow b^2 = \frac{784 - 441}{9} = \frac{343}{9}$

Thus, the equation of the hyperbola is $\frac{x^2}{49} - \frac{9y^2}{343} = 1$.

Question 15:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(0, \pm\sqrt{10}\right)$, passing through (2, 3)

Foci $\left(0, \pm\sqrt{10}\right)$, passing through (2, 3)

Here, the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Since the foci are $\left(0, \pm\sqrt{10}\right)$, $c = \sqrt{10}$.

We know that $a^2 + b^2 = c^2$.

$\therefore a^2 + b^2 = 10$

$\Rightarrow b^2 = 10 - a^2$ … (1)

Since the hyperbola passes through point (2, 3),

$$\frac{9}{a^2}-\frac{4}{b^2}=1 \qquad ...(2)$$

From equations (1) and (2), we obtain

$$\frac{9}{a^2}-\frac{4}{(10-a^2)}=1$$
$$\Rightarrow 9(10-a^2)-4a^2=a^2(10-a^2)$$
$$\Rightarrow 90-9a^2-4a^2=10a^2-a^4$$
$$\Rightarrow a^4-23a^2+90=0$$
$$\Rightarrow a^4-18a^2-5a^2+90=0$$
$$\Rightarrow a^2(a^2-18)-5(a^2-18)=0$$
$$\Rightarrow (a^2-18)(a^2-5)=0$$
$$\Rightarrow a^2=18 \text{ or } 5$$

In hyperbola, $c > a$, i.e., $c^2 > a^2$

$\therefore a^2 = 5$

$\Rightarrow b^2 = 10 - a^2 = 10 - 5 = 5$

Thus, the equation of the hyperbola is $\frac{y^2}{5}-\frac{x^2}{5}=1$.