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EXERCISE:- 4.1

Question 1:

Prove the following by using the principle of mathematical induction for all $n \in N$: $1+3+3^2+...+3^{n-1} = \frac{(3^n-1)}{2}$

Let the given statement be P(n), i.e.,

P(n): 1 + 3 + 3² + ... + 3ⁿ⁻¹ =
$$\frac{(3^n - 1)}{2}$$

For n = 1, we have

P(1): 1 =
$$\frac{(3^{1}-1)}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$$
, which is true.

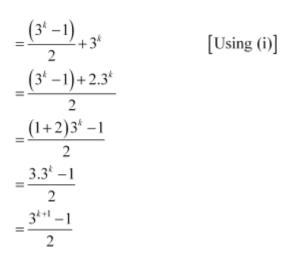
Let P(k) be true for some positive integer k, i.e.,

$$1 + 3 + 32 + ... + 3k-1 = \frac{(3k - 1)}{2} \qquad ...(i)$$

We shall now prove that P(k + 1) is true.

- $1 + 3 + 3^2 + \ldots + 3^{k-1} + 3^{(k+1)-1}$
- $= (1 + 3 + 3^2 + \ldots + 3^{k-1}) + 3^k$





Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 2:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
:
 $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)$

Let the given statement be P(n), i.e.,

P(n):
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

For n = 1, we have

P(1):
$$1^3 = 1 = \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{1.2}{2}\right)^2 = 1^2 = 1$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,



We shall now prove that P(k + 1) is true.

Consider

 $1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3$

$$= \left(\frac{k(k+1)}{2}\right)^{2} + (k+1)^{3} \qquad [Using (i)]$$

$$= \frac{k^{2}(k+1)^{2}}{4} + (k+1)^{3}$$

$$= \frac{k^{2}(k+1)^{2} + 4(k+1)^{3}}{4}$$

$$= \frac{(k+1)^{2} \left\{k^{2} + 4(k+1)\right\}}{4}$$

$$= \frac{(k+1)^{2} \left\{k^{2} + 4k + 4\right\}}{4}$$

$$= \frac{(k+1)^{2} \left\{k^{2} + 4k + 4\right\}}{4}$$

$$= \frac{(k+1)^{2} (k+2)^{2}}{4}$$

$$= \frac{(k+1)^{2} (k+1+1)^{2}}{4}$$

$$= \left(\frac{(k+1)(k+1+1)}{2}\right)^{2}$$

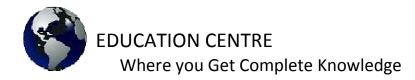
Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 3:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots n)} = \frac{2n}{(n+1)}$



Let the given statement be P(n), i.e.,

P(n):
$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots n} = \frac{2n}{n+1}$$

For n = 1, we have

P(1):
$$1 = \frac{2.1}{1+1} = \frac{2}{2} = 1$$
 which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1 + \frac{1}{1+2} + \dots + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+k} = \frac{2k}{k+1} \qquad \dots (i)$$

We shall now prove that P(k + 1) is true.

$$\begin{aligned} 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+k} + \frac{1}{1+2+3+\dots+k+(k+1)} \\ &= \left(1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+k}\right) + \frac{1}{1+2+3+\dots+k+(k+1)} \\ &= \frac{2k}{k+1} + \frac{1}{1+2+3+\dots+k+(k+1)} \\ &= \frac{2k}{k+1} + \frac{1}{\left(\frac{(k+1)(k+1+1)}{2}\right)} \\ &= \frac{2k}{(k+1)} + \frac{2}{(k+1)(k+2)} \\ &= \frac{2}{(k+1)} \left(k + \frac{1}{k+2}\right) \end{aligned}$$



$$= \frac{2}{(k+1)} \left(\frac{k^2 + 2k + 1}{k+2} \right)$$
$$= \frac{2}{(k+1)} \left(\frac{k^2 + 2k + 1}{k+2} \right)$$
$$= \frac{2 \cdot (k+1)^2}{(k+1)(k+2)}$$
$$= \frac{2(k+1)}{(k+2)}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 4:

Prove the following by using the principle of mathematical induction for all $n \in N$: 1.2.3 + 2.3.4 + ... + $n(n + 1)(n + 2) = \frac{n(n+1)(n+2)(n+3)}{4}$

$$+2.3.4+\ldots+n(n+1)(n+2)=$$

Let the given statement be P(n), i.e.,

P(n): 1.2.3 + 2.3.4 + ... + n(n + 1) (n + 2) =
$$\frac{n(n+1)(n+2)(n+3)}{4}$$

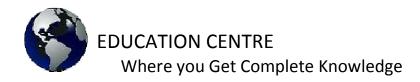
For n = 1, we have

P(1): 1.2.3 = 6 = $\frac{1(1+1)(1+2)(1+3)}{4} = \frac{1.2.3.4}{4} = 6$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{k(k+1)(k+2)(k+3)}{4} \dots (i)$$

We shall now prove that P(k + 1) is true.



$$1.2.3 + 2.3.4 + \dots + k(k+1)(k+2) + (k+1)(k+2)(k+3)$$
$$= \{1.2.3 + 2.3.4 + \dots + k(k+1)(k+2)\} + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3) \qquad [Using (i)]$$

= $(k+1)(k+2)(k+3)\left(\frac{k}{4}+1\right)$
= $\frac{(k+1)(k+2)(k+3)(k+4)}{4}$
= $\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 5:

Prove the following by using the principle of mathematical induction for

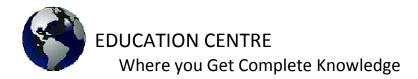
all
$$n \in N$$
:
 $1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$

Let the given statement be P(n), i.e.,

P(n):
$$1.3 + 2.3^2 + 3.3^3 + ... + n3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

For n = 1, we have

P(1): 1.3 = 3 =
$$\frac{(2.1-1)3^{1+1}+3}{4} = \frac{3^2+3}{4} = \frac{12}{4} = 3$$
, which is true.



Let P(k) be true for some positive integer k, i.e.,

$$1.3 + 2.3^{2} + 3.3^{3} + \dots + k3^{k} = \frac{(2k-1)3^{k+1} + 3}{4} \qquad \dots (i)$$

We shall now prove that P(k + 1) is true.

Consider

$$1.3 + 2.3^{2} + 3.3^{3} + \dots + k3^{k+} (k+1) 3^{k+1}$$

$$= (1.3 + 2.3^{2} + 3.3^{3} + \dots + k.3^{k}) + (k+1) 3^{k+1}$$

$$= \frac{(2k-1)3^{k+1} + 3}{4} + (k+1)3^{k+1}$$

$$= \frac{(2k-1)3^{k+1} + 3 + 4(k+1)3^{k+1}}{4}$$

$$= \frac{3^{k+1} \{2k-1+4(k+1)\} + 3}{4}$$

$$= \frac{3^{k+1} \{6k+3\} + 3}{4}$$

$$= \frac{3^{k+1} \{6k+3\} + 3}{4}$$

$$= \frac{3^{(k+1)+1} \{2k+1\} + 3}{4}$$

$$= \frac{3^{(k+1)+1} \{2k+1\} + 3}{4}$$

Thus, P(k + 1) is true whenever P(k) is true.

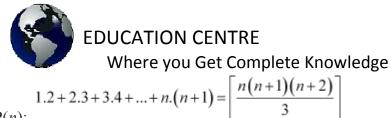
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 6:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
:
 $1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$

Let the given statement be P(n), i.e.,



P(*n*):

For n = 1, we have

P(1):
$$1.2 = 2 = \frac{1(1+1)(1+2)}{3} = \frac{1.2.3}{3} = 2$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1.2 + 2.3 + 3.4 + \dots + k.(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right] \dots (i)$$

We shall now prove that P(k + 1) is true.

Consider

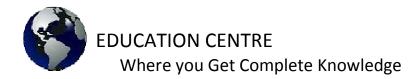
$$1.2 + 2.3 + 3.4 + \dots + k.(k + 1) + (k + 1).(k + 2)$$

= $[1.2 + 2.3 + 3.4 + \dots + k.(k + 1)] + (k + 1).(k + 2)$
= $\frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$ [Using (i)]
= $(k+1)(k+2)\left(\frac{k}{3}+1\right)$
= $\frac{(k+1)(k+2)(k+3)}{3}$
= $\frac{(k+1)(k+1+1)(k+1+2)}{3}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 7:



Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
:
 $1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Let the given statement be P(n), i.e.,

P(n):
$$1.3+3.5+5.7+...+(2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

For n = 1, we have

P(1):1.3 = 3 =
$$\frac{1(4.1^2 + 6.1 - 1)}{3} = \frac{4 + 6 - 1}{3} = \frac{9}{3} = 3$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1.3 + 3.5 + 5.7 + \dots + (2k-1)(2k+1) = \frac{k(4k^2 + 6k-1)}{3} \dots (i)$$

We shall now prove that P(k + 1) is true.

$$(1.3 + 3.5 + 5.7 + ... + (2k - 1)(2k + 1) + {2(k + 1) - 1} {2(k + 1) + 1}$$

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$$= \frac{k(4k^{2}+6k-1)}{3} + (2k+2-1)(2k+2+1) \qquad [Using (i)]$$

$$= \frac{k(4k^{2}+6k-1)}{3} + (2k+1)(2k+3)$$

$$= \frac{k(4k^{2}+6k-1)+3(4k^{2}+8k+3)}{3}$$

$$= \frac{4k^{3}+6k^{2}-k+12k^{2}+24k+9}{3}$$

$$= \frac{4k^{3}+18k^{2}+23k+9}{3}$$

$$= \frac{4k^{3}+14k^{2}+9k+4k^{2}+14k+9}{3}$$

$$= \frac{k(4k^{2}+14k+9)+1(4k^{2}+14k+9)}{3}$$

$$= \frac{(k+1)\{4k^{2}+8k+4+6k+6-1\}}{3}$$

$$= \frac{(k+1)\{4(k^{2}+2k+1)+6(k+1)-1\}}{3}$$

$$= \frac{(k+1)\{4(k+1)^{2}+6(k+1)-1\}}{3}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 8:

Prove the following by using the principle of mathematical induction for all $n \in N$: 1.2 + 2.2² + 3.2² + ... + $n.2^n = (n-1) 2^{n+1} + 2$

Let the given statement be P(n), i.e.,



P(n):
$$1.2 + 2.2^2 + 3.2^2 + ... + n.2^n = (n-1) 2^{n+1} + 2$$

For n = 1, we have

P(1): $1.2 = 2 = (1 - 1) 2^{1+1} + 2 = 0 + 2 = 2$, which is true.

Let P(k) be true for some positive integer k, i.e.,

 $1.2 + 2.2^2 + 3.2^2 + \ldots + k.2^k = (k-1) 2^{k+1} + 2 \ldots$ (i)

We shall now prove that P(k + 1) is true.

Consider

$$\{1.2 + 2.2^2 + 3.2^3 + \dots + k.2^k\} + (k+1) \cdot 2^{k+1}$$

= $(k-1)2^{k+1} + 2 + (k+1)2^{k+1}$
= $2^{k+1}\{(k-1) + (k+1)\} + 2$
= $2^{k+1}.2k + 2$
= $k.2^{(k+1)+1} + 2$
= $\{(k+1)-1\}2^{(k+1)+1} + 2$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

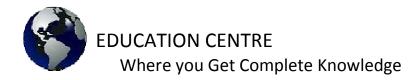
Question 9:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Let the given statement be P(n), i.e.,

P(n):
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$



For n = 1, we have

P(1): $\frac{1}{2} = 1 - \frac{1}{2^1} = \frac{1}{2}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{k}} = 1 - \frac{1}{2^{k}} \qquad \dots (i)$$

We shall now prove that P(k + 1) is true.

Consider

$$\begin{pmatrix} \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{k}} \end{pmatrix} + \frac{1}{2^{k+1}}$$

$$= \left(1 - \frac{1}{2^{k}}\right) + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^{k}} + \frac{1}{2 \cdot 2^{k}}$$

$$= 1 - \frac{1}{2^{k}} \left(1 - \frac{1}{2}\right)$$

$$= 1 - \frac{1}{2^{k}} \left(\frac{1}{2}\right)$$

$$= 1 - \frac{1}{2^{k+1}}$$

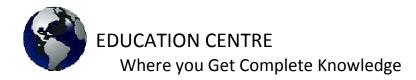
$$[Using (i)]$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 10:

Prove the following by using the principle of mathematical induction for all $n \in N$: $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$



Let the given statement be P(n), i.e.,

P(n):
$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$$

For n = 1, we have

 $P(1) = \frac{1}{2.5} = \frac{1}{10} = \frac{1}{6.1+4} = \frac{1}{10}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3k-1)(3k+2)} = \frac{k}{6k+4} \qquad \dots (i)$$

We shall now prove that P(k + 1) is true.

Consider

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3k-1)(3k+2)} + \frac{1}{\{3(k+1)-1\}\{3(k+1)+2\}}$$

$$= \frac{k}{6k+4} + \frac{1}{(3k+3-1)(3k+3+2)}$$
[Using (i)]
$$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)}$$

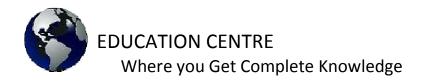
$$= \frac{1}{(3k+2)} \left(\frac{k}{2} + \frac{1}{3k+5}\right)$$

$$= \frac{1}{(3k+2)} \left(\frac{k(3k+5)+2}{2(3k+5)}\right)$$

$$= \frac{1}{(3k+2)} \left(\frac{3k^2 + 5k + 2}{2(3k+5)}\right)$$

$$= \frac{1}{(3k+2)} \left(\frac{(3k+2)(k+1)}{2(3k+5)}\right)$$

$$= \frac{(k+1)}{6k+10}$$



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 11:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

Let the given statement be P(n), i.e.,

P(n):
$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

For n = 1, we have

P(1):
$$\frac{1}{1 \cdot 2 \cdot 3} = \frac{1 \cdot (1+3)}{4(1+1)(1+2)} = \frac{1 \cdot 4}{4 \cdot 2 \cdot 3} = \frac{1}{1 \cdot 2 \cdot 3}$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \dots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)} \qquad \dots (i)$$

We shall now prove that P(k + 1) is true.

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$$\left[\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \dots + \frac{1}{k(k+1)(k+2)}\right] + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$
[Using (i)]

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k(k+3)^2 + 4}{4(k+3)} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k(k^2 + 6k + 9) + 4}{4(k+3)} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k^3 + 6k^2 + 9k + 4}{4(k+3)} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k^3 + 2k^2 + k + 4k^2 + 8k + 4}{4(k+3)} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k(k^2 + 2k + 1) + 4(k^2 + 2k + 1)}{4(k+3)} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \left\{ \frac{k(k+1)^2 + 4(k+1)^2}{4(k+3)} \right\}$$

$$= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}$$

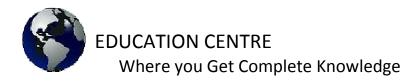
$$= \frac{(k+1)^2(k+4)}{4((k+1)+1)\{(k+1)+2\}}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 12:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
:
 $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$



Let the given statement be P(n), i.e.,

$$P(n): a + ar + ar^{2} + ... + ar^{n-1} = \frac{a(r^{n} - 1)}{r - 1}$$

For n = 1, we have

$$P(1): a = \frac{a(r^1 - 1)}{(r - 1)} = a$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$a + ar + ar^{2} + \dots + ar^{k-1} = \frac{a(r^{k} - 1)}{r - 1} \dots$$
(i)

We shall now prove that P(k + 1) is true.

Consider

$$\{a + ar + ar^{2} + \dots + ar^{k-1}\} + ar^{(k+1)-1}$$

$$= \frac{a(r^{k} - 1)}{r - 1} + ar^{k} \qquad [Using(i)]$$

$$= \frac{a(r^{k} - 1) + ar^{k}(r - 1)}{r - 1}$$

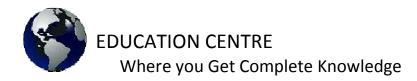
$$= \frac{a(r^{k} - 1) + ar^{k+1} - ar^{k}}{r - 1}$$

$$= \frac{ar^{k} - a + ar^{k+1} - ar^{k}}{r - 1}$$

$$= \frac{ar^{k+1} - a}{r - 1}$$

$$= \frac{a(r^{k+1} - 1)}{r - 1}$$

Thus, P(k + 1) is true whenever P(k) is true.



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 13:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right)...\left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$

Let the given statement be P(n), i.e.,

$$P(n):\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)...\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2$$

For n = 1, we have

P(1):
$$\left(1+\frac{3}{1}\right) = 4 = \left(1+1\right)^2 = 2^2 = 4$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

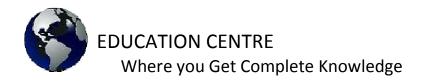
$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\dots\left(1+\frac{(2k+1)}{k^2}\right) = (k+1)^2 \qquad \dots (1)$$

We shall now prove that P(k + 1) is true.

Consider

$$\begin{bmatrix} \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)...\left(1+\frac{(2k+1)}{k^2}\right)\end{bmatrix} \left\{1+\frac{\left\{2(k+1)+1\right\}}{(k+1)^2}\right\}$$

= $(k+1)^2 \left(1+\frac{2(k+1)+1}{(k+1)^2}\right)$ [Using (1)]
= $(k+1)^2 \left[\frac{(k+1)^2+2(k+1)+1}{(k+1)^2}\right]$
= $(k+1)^2+2(k+1)+1$
= $\{(k+1)+1\}^2$



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 14:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)...\left(1 + \frac{1}{n}\right) = (n+1)$

Let the given statement be P(n), i.e.,

$$P(n):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{n}\right)=(n+1)$$

For n = 1, we have

 $P(1):(1+\frac{1}{1})=2=(1+1)$, which is true.

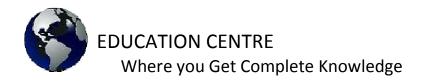
Let P(k) be true for some positive integer k, i.e.,

$$P(k):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{k}\right)=(k+1) \qquad ... (1)$$

We shall now prove that P(k + 1) is true.

Consider

$$\begin{bmatrix} \left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{k}\right)\end{bmatrix}\left(1+\frac{1}{k+1}\right) \\ = (k+1)\left(1+\frac{1}{k+1}\right) \\ = (k+1)\left(\frac{(k+1)+1}{(k+1)}\right) \\ = (k+1)+1 \end{bmatrix}$$
[Using (1)]



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 15:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
:
 $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Let the given statement be P(n), i.e.,

$$P(n) = 1^{2} + 3^{2} + 5^{2} + \dots + (2n-1)^{2} = \frac{n(2n-1)(2n+1)}{3}$$

For n = 1, we have

$$P(1) = 1^2 = 1 = \frac{1(2.1-1)(2.1+1)}{3} = \frac{1.1.3}{3} = 1$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$P(k) = 1^{2} + 3^{2} + 5^{2} + \dots + (2k-1)^{2} = \frac{k(2k-1)(2k+1)}{3} \qquad \dots (1)$$

We shall now prove that P(k + 1) is true.

Consider

$$\begin{cases} 1^{2} + 3^{2} + 5^{2} + \dots + (2k-1)^{2} \} + \{2(k+1)-1\}^{2} \\ = \frac{k(2k-1)(2k+1)}{3} + (2k+2-1)^{2} \\ = \frac{k(2k-1)(2k+1)}{3} + (2k+1)^{2} \\ = \frac{k(2k-1)(2k+1) + 3(2k+1)^{2}}{3} \\ = \frac{(2k+1)\{k(2k-1)+3(2k+1)\}}{3} \\ = \frac{(2k+1)\{2k^{2}-k+6k+3\}}{3} \end{cases}$$

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$$=\frac{(2k+1)\{2k^{2}+5k+3\}}{3}$$

$$=\frac{(2k+1)\{2k^{2}+2k+3k+3\}}{3}$$

$$=\frac{(2k+1)\{2k(k+1)+3(k+1)\}}{3}$$

$$=\frac{(2k+1)(k+1)(2k+3)}{3}$$

$$=\frac{(k+1)\{2(k+1)-1\}\{2(k+1)+1\}}{3}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 16:

Prove the following by using the principle of mathematical induction for

all $n \in N$: $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$

Let the given statement be P(n), i.e.,

$$P(n): \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

For n = 1, we have

 $P(1) = \frac{1}{1.4} = \frac{1}{3.1+1} = \frac{1}{4} = \frac{1}{1.4}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$P(k) = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \qquad \dots (1)$$

We shall now prove that P(k + 1) is true.

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$$\left\{\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3k-2)(3k+1)}\right\} + \frac{1}{\{3(k+1)-2\}\{3(k+1)+1\}}$$

$$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$$
[Using (1)]

$$= \frac{1}{(3k+1)} \left\{k + \frac{1}{(3k+4)}\right\}$$

$$= \frac{1}{(3k+1)} \left\{\frac{k(3k+4)+1}{(3k+4)}\right\}$$

$$= \frac{1}{(3k+1)} \left\{\frac{3k^2 + 4k + 1}{(3k+4)}\right\}$$

$$= \frac{1}{(3k+1)} \left\{\frac{3k^2 + 3k + k + 1}{(3k+4)}\right\}$$

$$= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)}{3(k+1)+1}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 17:

Prove the following by using the principle of mathematical induction for

all
$$n \in N$$
: $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

Let the given statement be P(n), i.e.,

$$P(n):\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

For n = 1, we have



$$P(1): \frac{1}{3.5} = \frac{1}{3(2.1+3)} = \frac{1}{3.5}$$
, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$P(k):\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \qquad \dots (1)$$

We shall now prove that P(k + 1) is true.

Consider

$$\begin{bmatrix} \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} \end{bmatrix} + \frac{1}{\{2(k+1)+1\}} \{2(k+1)+3\}}$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \qquad [Using (1)]$$

$$= \frac{1}{(2k+3)} \begin{bmatrix} \frac{k}{3} + \frac{1}{(2k+5)} \end{bmatrix}$$

$$= \frac{1}{(2k+3)} \begin{bmatrix} \frac{k(2k+5)+3}{3(2k+5)} \end{bmatrix}$$

$$= \frac{1}{(2k+3)} \begin{bmatrix} \frac{2k^2+5k+3}{3(2k+5)} \end{bmatrix}$$

$$= \frac{1}{(2k+3)} \begin{bmatrix} \frac{2k^2+2k+3k+3}{3(2k+5)} \end{bmatrix}$$

$$= \frac{1}{(2k+3)} \begin{bmatrix} \frac{2k(k+1)+3(k+1)}{3(2k+5)} \end{bmatrix}$$

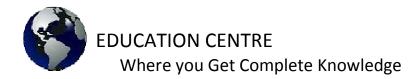
$$= \frac{(k+1)(2k+3)}{3(2k+5)}$$

$$= \frac{(k+1)}{3\{2(k+1)+3\}}$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 18:



Prove the following by using the principle of mathematical induction for all $n \in N$: $1+2+3+...+n < \frac{1}{8}(2n+1)^2$

Let the given statement be P(n), i.e.,

$$P(n): 1+2+3+...+n < \frac{1}{8}(2n+1)^2$$

It can be noted that P(n) is true for n = 1 since $1 < \frac{1}{8}(2.1+1)^2 = \frac{9}{8}$.

Let P(k) be true for some positive integer k, i.e.,

$$1+2+...+k < \frac{1}{8}(2k+1)^2$$
 (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$$(1+2+...+k)+(k+1) < \frac{1}{8}(2k+1)^{2}+(k+1) \qquad [Using(1)]$$

$$<\frac{1}{8}\{(2k+1)^{2}+8(k+1)\}$$

$$<\frac{1}{8}\{4k^{2}+4k+1+8k+8\}$$

$$<\frac{1}{8}\{4k^{2}+12k+9\}$$

$$<\frac{1}{8}\{2(k+3)^{2}$$

$$<\frac{1}{8}\{2(k+1)+1\}^{2}$$

Hence, $(1+2+3+...+k)+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$

Thus, P(k + 1) is true whenever P(k) is true.



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 19:

Prove the following by using the principle of mathematical induction for all $n \in N$: n (n + 1) (n + 5) is a multiple of 3.

Let the given statement be P(n), i.e.,

P(n): n(n + 1)(n + 5), which is a multiple of 3.

It can be noted that P(n) is true for n = 1 since 1(1 + 1)(1 + 5) = 12, which is a multiple of 3.

Let P(k) be true for some positive integer k, i.e.,

k(k+1)(k+5) is a multiple of 3.

: k(k+1)(k+5) = 3m, where $m \in \mathbb{N}$... (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

$$(k+1)\{(k+1)+1\}\{(k+1)+5\}$$

$$= (k+1)(k+2)\{(k+5)+1\}$$

$$= (k+1)(k+2)(k+5)+(k+1)(k+2)$$

$$= \{k(k+1)(k+5)+2(k+1)(k+5)\}+(k+1)(k+2)$$

$$= 3m + (k+1)\{2(k+5)+(k+2)\}$$

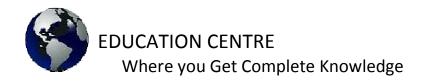
$$= 3m + (k+1)\{2k+10+k+2\}$$

$$= 3m + (k+1)(3k+12)$$

$$= 3m + (k+1)(3k+12)$$

$$= 3m + (k+1)(k+4)$$

$$= 3\{m + (k+1)(k+4)\} = 3 \times q, \text{ where } q = \{m + (k+1)(k+4)\} \text{ is some natural number Therefore, } (k+1)\{(k+1)+1\}\{(k+1)+5\} \text{ is a multiple of 3.}$$



Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 20:

Prove the following by using the principle of mathematical induction for all $n \in N$: $10^{2n-1} + 1$ is divisible by 11.

Let the given statement be P(n), i.e.,

P(*n*): $10^{2n-1} + 1$ is divisible by 11.

It can be observed that P(n) is true for n = 1 since $P(1) = 10^{2.1-1} + 1 = 11$, which is divisible by 11.

Let P(k) be true for some positive integer k, i.e.,

 $10^{2k-1} + 1$ is divisible by 11.

 $::10^{2k-1} + 1 = 11m$, where $m \in \mathbb{N}$... (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.



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10^{2(k+1)-1} + 1
= 10^{2k+2-1} + 1
= 10^{2(k+1)} + 1
= 10^{2} (10^{2k-1} + 1 - 1) + 1
= 10^{2} (10^{2k-1} + 1) - 10^{2} + 1
= 10^{2} .11m - 100 + 1 [Using (1)]
= 100 \times 11m - 99
= 11(100m - 9)
= 11r, where r = (100m - 9) is some natural number
Therefore, 10^{2(k+1)-1} + 1 is divisible by 11.
```

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 21:

Prove the following by using the principle of mathematical induction for all $n \in N$: $x^{2n} - y^{2n}$ is divisible by x + y.

Let the given statement be P(n), i.e.,

P(*n*): $x^{2n} - y^{2n}$ is divisible by x + y.

It can be observed that P(n) is true for n = 1.

This is so because $x^{2 \times 1} - y^{2 \times 1} = x^2 - y^2 = (x + y) (x - y)$ is divisible by (x + y).

Let P(k) be true for some positive integer k, i.e.,

 $x^{2k} - y^{2k}$ is divisible by x + y.

 $\therefore x^{2k} - y^{2k} = m (x + y)$, where $m \in \mathbb{N} \dots (1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.



$$\begin{aligned} x^{2(x+y)} &- y^{2(x+y)} \\ &= x^{2k} \cdot x^2 - y^{2k} \cdot y^2 \\ &= x^2 \left(x^{2k} - y^{2k} + y^{2k} \right) - y^{2k} \cdot y^2 \\ &= x^2 \left\{ m(x+y) + y^{2k} \right\} - y^{2k} \cdot y^2 \qquad \left[\text{Using (1)} \right] \\ &= m(x+y)x^2 + y^{2k} \cdot x^2 - y^{2k} \cdot y^2 \\ &= m(x+y)x^2 + y^{2k} \left(x^2 - y^2 \right) \\ &= m(x+y)x^2 + y^{2k} \left(x^2 - y^2 \right) \\ &= m(x+y)x^2 + y^{2k} \left(x + y \right) (x-y) \\ &= (x+y) \left\{ mx^2 + y^{2k} \left(x - y \right) \right\}, \text{ which is a factor of } (x+y). \end{aligned}$$

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 22:

Prove the following by using the principle of mathematical induction for all $n \in N$: $3^{2n+2} - 8n - 9$ is divisible by 8.

Let the given statement be P(n), i.e.,

P(*n*): $3^{2n+2} - 8n - 9$ is divisible by 8.

It can be observed that P(n) is true for n = 1 since $3^{2 \times 1+2} - 8 \times 1 - 9 = 64$, which is divisible by 8.

Let P(k) be true for some positive integer k, i.e.,

 $3^{2k+2} - 8k - 9$ is divisible by 8.

 $::3^{2k+2} - 8k - 9 = 8m$; where $m \in \mathbb{N}$... (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.



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$$3^{2(k+1)+2} - 8(k+1) - 9$$

= $3^{2k+2} \cdot 3^2 - 8k - 8 - 9$
= $3^2 (3^{2k+2} - 8k - 9 + 8k + 9) - 8k - 17$
= $3^2 (3^{2k+2} - 8k - 9) + 3^2 (8k + 9) - 8k - 17$
= $9.8m + 9(8k + 9) - 8k - 17$
= $9.8m + 72k + 81 - 8k - 17$
= $9.8m + 64k + 64$
= $8(9m + 8k + 8)$
= $8r$, where $r = (9m + 8k + 8)$ is a natural number
Therefore, $3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8.

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 23:

Prove the following by using the principle of mathematical induction for all $n \in N$: $41^n - 14^n$ is a multiple of 27.

Let the given statement be P(n), i.e.,

 $P(n):41^{n} - 14^{n}$ is a multiple of 27.

It can be observed that P(n) is true for n = 1 since $41^{1} - 14^{1} = 27$, which is a multiple of 27.

Let P(k) be true for some positive integer k, i.e.,

 $41^{k} - 14^{k}$ is a multiple of 27

 $::41^{k} - 14^{k} = 27m$, where $m \in \mathbb{N} \dots (1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.



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$$41^{k+1} - 14^{k+1}$$

= $41^{k} \cdot 41 - 14^{k} \cdot 14$
= $41(41^{k} - 14^{k} + 14^{k}) - 14^{k} \cdot 14$
= $41(41^{k} - 14^{k}) + 41.14^{k} - 14^{k} \cdot 14$
= $41.27m + 14^{k} (41 - 14)$
= $41.27m + 27.14^{k}$
= $27(41m - 14^{k})$
= $27 \times r$, where $r = (41m - 14^{k})$ is a natural number
Therefore, $41^{k+1} - 14^{k+1}$ is a multiple of 27.

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Question 24:

Prove the following by using the principle of mathematical induction for all $n \in N$:

 $(2n+7) < (n+3)^2$

Let the given statement be P(n), i.e.,

P(*n*): $(2n+7) < (n+3)^2$

It can be observed that P(n) is true for n = 1 since $2 \cdot 1 + 7 = 9 < (1 + 3)^2 = 16$, which is true.

Let P(k) be true for some positive integer k, i.e.,

 $(2k+7) < (k+3)^2 \dots (1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.

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$$\{2(k+1)+7\} = (2k+7)+2$$

 $\therefore \{2(k+1)+7\} = (2k+7)+2 < (k+3)^2 + 2$ [u sing (1)]
 $2(k+1)+7 < k^2 + 6k + 9 + 2$
 $2(k+1)+7 < k^2 + 6k + 11$
Now, $k^2 + 6k + 11 < k^2 + 8k + 16$
 $\therefore 2(k+1)+7 < (k+4)^2$
 $2(k+1)+7 < {(k+1)+3}^2$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.