EXERCISE:-12.1

Question 1:

A point is on the x-axis. What are its y-coordinates and z-coordinates?

If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

Question 2:

A point is in the XZ-plane. What can you say about its y-coordinate?

If a point is in the XZ plane, then its y-coordinate is zero.

Question 3:

Name the octants in which the following points lie:

(1, 2, 3), (4, –2, 3), (4, –2, –5), (4, 2, –5), (–4, 2, –5), (–4, 2, 5),

(–3, –1, 6), (2, –4, –7)

The x-coordinate, y-coordinate, and z-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, –2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, –2, –5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, 2, –5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The x-coordinate, y-coordinate, and z-coordinate of point (–4, 2, –5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The x-coordinate, y-coordinate, and z-coordinate of point (–4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

The x-coordinate, y-coordinate, and z-coordinate of point (–3, –1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant **III**.

The x-coordinate, y-coordinate, and z-coordinate of point (2, –4, –7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Question 4:

Fill in the blanks:

(i) The x-axis and y-axis taken together determine a plane known as $\underline{\text{XY} - \text{plane}}$.

(ii) The coordinates of points in the XY-plane are of the form $\underline{(x, y, 0)}$.

(iii) Coordinate planes divide the space into $\underline{\text{eight}}$ octants.

EXERCISE:-12.2

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4) (iv) (2, –1, 3) and (–2, 1, 3)

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by $\text{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$=\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$=\sqrt{(2)^2+(0)^2+(-4)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

$$=\sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$$
$$=\sqrt{(5)^2+(-3)^2+(-3)^2}$$
$$=\sqrt{25+9+9}$$
$$=\sqrt{43}$$

(iii) Distance between points (–1, 3, –4) and (1, –3, 4)

$$=\sqrt{(1+1)^2+(-3-3)^2+(4+4)^2}$$
$$=\sqrt{(2)^2+(-6)^3+(8)^2}$$
$$=\sqrt{4+36+64}=\sqrt{104}=2\sqrt{26}$$

(iv) Distance between points (2, –1, 3) and (–2, 1, 3)

$$=\sqrt{(-2-2)^2+(1+1)^2+(3-3)^2}$$
$$=\sqrt{(-4)^2+(2)^2+(0)^2}$$
$$=\sqrt{16+4}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Question 2:

Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Let points (–2, 3, 5), (1, 2, 3), and (7, 0, –1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$
\begin{aligned}
PQ &= \sqrt{(1+2)^2+(2-3)^2+(3-5)^2} \\
&= \sqrt{(3)^2+(-1)^2+(-2)^2} \\
&= \sqrt{9+1+4} \\
&= \sqrt{14}
\end{aligned}
$$

$$
\begin{aligned}
QR &= \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2} \\
&= \sqrt{(6)^2+(-2)^2+(-4)^2} \\
&= \sqrt{36+4+16} \\
&= \sqrt{56} \\
&= 2\sqrt{14}
\end{aligned}
$$

$$
\begin{aligned}
PR &= \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2} \\
&= \sqrt{(9)^2+(-3)^2+(-6)^2} \\
&= \sqrt{81+9+36} \\
&= \sqrt{126} \\
&= 3\sqrt{14}
\end{aligned}
$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(–2, 3, 5), Q(1, 2, 3), and R(7, 0, –1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, –10), (1, 6, –6) and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (–4, 9, 6) are the vertices of a right angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

(i) Let points (0, 7, –10), (1, 6, –6), and (4, 9, –6) be denoted by A, B, and C respectively.

$$
\begin{aligned}
AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\
&= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\
&= \sqrt{1+1+16} \\
&= \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\
&= \sqrt{(3)^2 + (3)^2} \\
&= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\
&= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\
&= \sqrt{16+4+16} = \sqrt{36} = 6
\end{aligned}
$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

$$
\begin{aligned}
AB &= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \\
&= \sqrt{(-1)^2 + (-1)^2 + (-4)^2} \\
&= \sqrt{1+1+16} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} \\
&= \sqrt{(-3)^2 + (3)^2 + (0)^2} \\
&= \sqrt{9+9} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$\begin{aligned} CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2} \\ &= \sqrt{(4)^2+(-2)^2+(4)^2} \\ &= \sqrt{16+4+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

Now, $AB^2+BC^2=\left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2=18+18=36=AC^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

$$\begin{aligned} AB &= \sqrt{(1+1)^2+(-2-2)^2+(5-1)^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2+(-7+2)^2+(8-5)^2} \\ &= \sqrt{9+25+9}=\sqrt{43} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(2-4)^2+(-3+7)^2+(4-8)^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(-1-2)^2+(2+3)^2+(1-4)^2} \\ &= \sqrt{9+25+9}=\sqrt{43} \end{aligned}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, –1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$
$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (–4, 0, 0) is equal to 10.

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$$
$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}=10-\sqrt{(x+4)^2+y^2+z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2+y^2+z^2=100-20\sqrt{(x+4)^2+y^2+z^2}+(x+4)^2+y^2+z^2$$
$$\Rightarrow x^2-8x+16+y^2+z^2=100-20\sqrt{x^2+8x+16+y^2+z^2}+x^2+8x+16+y^2+z^2$$
$$\Rightarrow 20\sqrt{x^2+8x+16+y^2+z^2}=100+16x$$
$$\Rightarrow 5\sqrt{x^2+8x+16+y^2+z^2}=(25+4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

EXERCISE:-12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio $m: n$ are

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(–2, 3, 5) and (1, –4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3},\ y=\frac{2(-4)+3(3)}{2+3},\ \text{and } z=\frac{2(6)+3(5)}{2+3}$$
$$\text{i.e., } x=\frac{-4}{5},\ y=\frac{1}{5},\ \text{and } z=\frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio m: n are

$$\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right).$$

Let R (x, y, z) be the point that divides the line segment joining points(–2, 3, 5) and (1, –4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3},\ y=\frac{2(-4)-3(3)}{2-3},\ \text{and } z=\frac{2(6)-3(5)}{2-3}$$
$$\text{i.e., } x=-8,\ y=17,\ \text{and } z=3$$

Thus, the coordinates of the required point are (–8, 17, 3).

Question 2:

Given that P (3, 2, –4), Q (5, 4, –6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Let point Q (5, 4, –6) divide the line segment joining points P (3, 2, –4) and R (9, 8, –10) in the ratio k:1.

Therefore, by section formula,

$$(5,4,-6)=\left(\frac{k(9)+3}{k+1},\frac{k(8)+2}{k+1},\frac{k(-10)-4}{k+1}\right)$$

$\Rightarrow \frac{9k+3}{k+1} = 5$
$\Rightarrow 9k+3 = 5k+5$
$\Rightarrow 4k = 2$
$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Let the YZ planedivide the line segment joining points (–2, 4, 7) and (3, –5, 8) in the ratio k:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$\frac{3k-2}{k+1} = 0$
$\Rightarrow 3k-2 = 0$
$\Rightarrow k = \frac{2}{3}$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4:

Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

The given points are A (2, –3, 4), B (–1, 2, 1), and $C\left(0, \frac{1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio k:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.

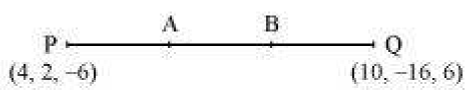
i.e., $C\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, –6) and Q (10, –16, 6).

Let A and B be the points that trisect the line segment joining points P (4, 2, –6) and Q (10, –16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2}\right) = (6, -4, -2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1},\frac{2(-16)+1(2)}{2+1},\frac{2(6)-1(6)}{2+1}\right)=(8,-10,2)$$

Thus, (6, –4, –2) and (8, –10, 2) are the points that trisect the line segment joining points P (4, 2, –6) and Q (10, –16, 6).