EXERCISE:-15.1

Question 1:

Find the mean deviation about the mean for the data

4, 7, 8, 9, 10, 12, 13, 17

The given data is

4, 7, 8, 9, 10, 12, 13, 17

Mean of the data, $\bar{x} = \dfrac{4+7+8+9+10+12+13+17}{8} = \dfrac{80}{8} = 10$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$, are

–6, – 3, –2, –1, 0, 2, 3, 7

The absolute values of the deviations, i.e. $|x_i - \bar{x}|$, are

6, 3, 2, 1, 0, 2, 3, 7

The required mean deviation about the mean is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{8}|x_i - \bar{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

Question 2:

Find the mean deviation about the mean for the data

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

The given data is

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Mean of the given data,

$$\bar{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

The deviations of the respective observations from the mean $\bar{x}$, i.e. $x_i - \bar{x}$, are

–12, 20, –2, –10, –8, 5, 13, –4, 4, –6

The absolute values of the deviations, i.e. $\left|x_i - \bar{x}\right|$, are

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

The required mean deviation about the mean is

$$\begin{aligned} \text{M.D.}(\bar{x}) &= \frac{\sum_{i=1}^{10}\left|x_i - \bar{x}\right|}{10} \\ &= \frac{12+20+2+10+8+5+13+4+4+6}{10} \\ &= \frac{84}{10} \\ &= 8.4 \end{aligned}$$

Question 3:

Find the mean deviation about the median for the data.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

The given data is

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Here, the numbers of observations are 12, which is even.

Arranging the data in ascending order, we obtain

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{Median, M} = \frac{\left(\frac{12}{2}\right)^{th} \text{observation} + \left(\frac{12}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$

$$= \frac{13+14}{2} = \frac{27}{2} = 13.5$$

The deviations of the respective observations from the median, i.e. $x_i - \text{M}$, are

–3.5, –2.5, –2.5, –1.5, –0.5, –0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The absolute values of the deviations, $|x_i - \text{M}|$, are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The required mean deviation about the median is

$$\text{M.D.}(\text{M}) = \frac{\sum_{i=1}^{12} |x_i - \text{M}|}{12}$$

$$= \frac{3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.5}{12}$$

$$= \frac{28}{12} = 2.33$$

Question 4:

Find the mean deviation about the median for the data

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

The given data is

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Here, the number of observations is 10, which is even.

Arranging the data in ascending order, we obtain

36, 42, 45, 46, 46, 49, 51, 53, 60, 72

$$\text{Median M} = \frac{\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{5^{th}\text{observation} + 6^{th}\text{observation}}{2}$$

$$= \frac{46+49}{2} = \frac{95}{2} = 47.5$$

The deviations of the respective observations from the median, i.e. $x_i - \text{M},$ are

–11.5, –5.5, –2.5, –1.5, –1.5, 1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the deviations, $|x_i - \text{M}|$, are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Thus, the required mean deviation about the median is

$$\text{M.D.}(\text{M}) = \frac{\sum_{i=1}^{10}|x_i - \text{M}|}{10} = \frac{11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.5}{10}$$

$$= \frac{70}{10} = 7$$

Question 5:

Find the mean deviation about the mean for the data.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6

20	3	60	6	18
25	5	125	11	55
	25	350		158

$$N = \sum_{i=1}^{5} f_i = 25$$

$$\sum_{i=1}^{5} f_i x_i = 350$$

$$\therefore \overline{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

$$\therefore MD(\overline{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i \left|x_i - \overline{x}\right| = \frac{1}{25} \times 158 = 6.32$$

Question 6:

Find the mean deviation about the mean for the data

x_i	10	30	50	70	90
f_i	4	24	28	16	8

x_i	f_i	$f_i x_i$	$\left\|x_i - \overline{x}\right\|$	$f_i \left\|x_i - \overline{x}\right\|$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$$N=\sum_{i=1}^{5} f_i = 80,\ \sum_{i=1}^{5} f_i x_i = 4000$$

$$\therefore\ \bar{x}=\frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{80}\times 4000 = 50$$

$$MD(\bar{x})\frac{1}{N}\sum_{i=1}^{5} f_i \left|x_i-\bar{x}\right| = \frac{1}{80}\times 1280 = 16$$

Question 7:

Find the mean deviation about the median for the data.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

x_i	f_i	*c.f.*
5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Here, N = 26, which is even.

Median is the mean of 13^{th} and 14^{th} observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

$$\therefore \text{Median} = \frac{13^{th}\text{ observation} + 14^{th}\text{ observation}}{2} = \frac{7+7}{2} = 7$$

The absolute values of the deviations from median, i.e. $|x_i - \text{M}|$, are

$\lvert x_i - \text{M}\rvert$	2	0	2	3	5	8
f_i	8	6	2	2	2	6
$f_i \lvert x_i - \text{M}\rvert$	16	0	4	6	10	48

$\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i |x_i - \text{M}| = 84$

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}}\sum_{i=1}^{6} f_i |x_i - \text{M}| = \frac{1}{26} \times 84 = 3.23$$

Question 8:

Find the mean deviation about the median for the data

x_i	15	21	27	30	35
f_i	3	5	6	7	8

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

x_i	f_i	c.f.
15	3	3
21	5	8
27	6	14

30	7	21
35	8	29

Here, N = 29, which is odd.

$\therefore$ Median $= \left(\frac{29+1}{2}\right)^{th}$ observation = 15th observation

This observation lies in the cumulative frequency 21, for which the corresponding observation is 30.

$\therefore$ Median = 30

The absolute values of the deviations from median, i.e. $|x_i - M|$, are

$\lvert x_i - M \rvert$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i \lvert x_i - M \rvert$	45	45	18	0	40

$$\sum_{i=1}^{5} f_i = 29, \ \sum_{i=1}^{5} f_i |x_i - M| = 148$$

$$\therefore \ \text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - M| = \frac{1}{29} \times 148 = 5.1$$

Question 9:

Find the mean deviation about the mean for the data.

Income per day	Number of persons
0-100	4
100-200	8
200-300	9

300-400	10
400-500	7
500-600	5
600-700	4
700-800	3

The following table is formed.

Income per day	**Number of persons f_i**	**Mid-point x_i**	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1168
700 – 800	3	750	2250	392	1176
	50		17900		7896

Here, $N = \sum_{i=1}^{8} f_i = 50, \ \sum_{i=1}^{8} f_i x_i = 17900$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i \lvert x_i - \bar{x} \rvert = \frac{1}{50} \times 7896 = 157.92$$

Question 10:

Find the mean deviation about the mean for the data

Height in cms	Number of boys
95-105	9
105-115	13
115-125	26
125-135	30
135-145	12
145-155	10

The following table is formed.

Height in cms	Number of boys f_i	Mid-point x_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247

Here, $N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}| = \frac{1}{100} \times 1128.8 = 11.28$$

Question 11:

Find the mean deviation about median for the following data:

Marks	Number of girls
0-10	6
10-20	8
20-30	14
30-40	16
40-50	4
50-60	2

The following table is formed.

Marks	Number of boys f_i	Cumulative frequency (c.f.)	Mid-point x_i	$\lvert x_i -$ Med.$\rvert$	$f_i \lvert x_i -$ Med.$\rvert$
0-10	6	6	5	22.85	137.1
10-20	8	14	15	12.85	102.8
20-30	14	28	25	2.85	39.9
30-40	16	44	35	7.15	114.4
40-50	4	48	45	17.15	68.6
50-60	2	50	55	27.15	54.3
	50				517.1

The class interval containing the $\left(\frac{N}{2}\right)^{th}$ or 25th item is 20 – 30.

Therefore, 20 – 30 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, $l = 20$, $C = 14$, $f = 14$, $h = 10$, and $N = 50$

$$\therefore \text{Median} = 20 + \frac{25-14}{14} \times 10 = 20 + \frac{110}{14} = 20 + 7.85 = 27.85$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i \left|x_i - M\right| = \frac{1}{50} \times 517.1 = 10.34$$

Question 12:

Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age	Number
16-20	5
21-25	6
26-30	12
31-35	14
36-40	26
41-45	12

46-50	16
51-55	9

The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

The table is formed as follows.

Age	**Number f_i**	**Cumulative frequency (c.f.)**	**Mid-point x_i**	$\lvert x_i -$ **Med.** $\rvert$	$f_i \lvert x_i -$ **Med.** $\rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	100				735

The class interval containing the $\frac{N}{2}^{th}$ or 50th item is 35.5 – 40.5.

Therefore, 35.5 – 40.5 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, $l = 35.5$, $C = 37$, $f = 26$, $h = 5$, and $N = 100$

$$\therefore \text{Median} = 35.5 + \frac{50-37}{26} \times 5 \quad = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(M) = \frac{1}{N} \sum_{i=1}^{8} f_i |x_i - M| = \frac{1}{100} \times 735 = 7.35$$

EXERCISE:-15.2

Question 1:

Find the mean and variance for the data 6, 7, 10, 12, 13, 4, 8, 12

6, 7, 10, 12, 13, 4, 8, 12

Mean, $\bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	–3	9

7	–2	4
10	–1	1
12	3	9
13	4	16
4	–5	25
8	–1	1
12	3	9
		74

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8}\times 74 = 9.25$$

Question 2:

Find the mean and variance for the first *n* natural numbers

The mean of first *n* natural numbers is calculated as follows.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{Variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i - \left(\frac{n+1}{2}\right)\right]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{i=1}^{n} 2\left(\frac{n+1}{2}\right)x_i + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{n}\right)\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

Question 3:

Find the mean and variance for the first 10 multiples of 3

The first 10 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here, number of observations, $n = 10$

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	–13.5	182.25
6	–10.5	110.25
9	–7.5	56.25
12	–4.5	20.25
15	–1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 742.5 = 74.25$$

Question 4:

Find the mean and variance for the data

xi	6	10	14	18	24	28	30
$f\,i$	2	4	7	12	8	4	3

The data is obtained in tabular form as follows.

x_i	$f\,i$	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$

6	2	12	–13	169	338
10	4	40	–9	81	324
14	7	98	–5	25	175
18	12	216	–1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	40	760			1736

Here, N = 40, $\sum_{i=1}^{7} f_i x_i = 760$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{760}{40} = 19$$

$$\text{Variance} = \left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

Question 5:

Find the mean and variance for the data

xi	92	93	97	98	102	104	109
f i	3	2	3	2	6	3	3

The data is obtained in tabular form as follows.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
92	3	276	–8	64	192
93	2	186	–7	49	98
97	3	291	–3	9	27

98	2	196	–2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

Here, N = 22, $\sum_{i=1}^{7} f_i x_i = 2200$

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

Question 6:

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

The data is obtained in tabular form as follows.

x_i	f_i	$f_i = \frac{x_i - 64}{1}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	–4	16	–8	32
61	1	–3	9	–3	9
62	12	–2	4	–24	48
63	29	–1	1	–29	29

64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

Mean, $\bar{x} = A \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$
$$= \frac{1}{100^2}\left[100 \times 286 - 0\right]$$
$$= 2.86$$

$\therefore$ Standard deviation $(\sigma) = \sqrt{2.86} = 1.69$

Question 7:

Find the mean and variance for the following frequency distribution.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	–3	9	–6	18
30-60	3	45	–2	4	–6	12
60-90	5	75	–1	1	–5	5

90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$

$$\text{Variance}\left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{(30)^2}{(30)^2}\left[30 \times 76 - (2)^2\right]$$

$$= 2280 - 4$$

$$= 2276$$

Question 8:

Find the mean and variance for the following frequency distribution.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i -}{1}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	–2	4	–10	20
10-	8	15	–1	1	–8	8

20						
20 - 30	15	25	0	0	0	0
30 - 40	16	35	1	1	16	16
40 - 50	6	45	2	4	12	24
	50				10	68

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}\left[50 \times 68 - (10)^2\right]$$

$$= \frac{1}{25}\left[3400 - 100\right] = \frac{3300}{25}$$

$$= 132$$

Question 9:

Find the mean, variance and standard deviation using short-cut method

Height in cms	No. of children
70-75	3

75-80	4
80-85	7
85-90	7
90-95	15
95-100	9
100-105	6
105-110	6
110-115	3

Class Interval	**Frequency** f_i	**Mid-point** x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	–4	16	–12	48
75-80	4	77.5	–3	9	–12	36
80-85	7	82.5	–2	4	–14	28
85-90	7	87.5	–1	1	–7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$

$$\text{Variance}\left(\sigma^2\right)=\frac{h^2}{N^2}\left[N\sum_{i=1}^{9}f_iy_i^{\,2}-\left(\sum_{i=1}^{9}f_iy_i\right)^2\right]$$

$$=\frac{(5)^2}{(60)^2}\left[60\times254-(6)^2\right]$$

$$=\frac{25}{3600}(15204)=105.58$$

$$\therefore \text{Standard deviation}(\sigma)=\sqrt{105.58}=10.27$$

Question 10:

The diameters of circles (in mm) drawn in a design are given below:

Diameters	No. of children
33-36	15
37-40	17
41-44	21
45-48	22
49-52	25

Class Interval	Frequency f_i	Mid-point x_i	$y_i=\frac{x_i-42.5}{4}$	f_i^2	fy_i	fy_i^2
32.5-36.5	15	34.5	–2	4	–30	60
36.5-40.5	17	38.5	–1	1	–17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Here, N = 100, h = 4

Let the assumed mean, A, be 42.5.

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$
$$= \frac{16}{10000}\left[100 \times 199 - (25)^2\right]$$
$$= \frac{16}{10000}[19900 - 625]$$
$$= \frac{16}{10000} \times 19275$$
$$= 30.84$$

$\therefore \text{Standard deviation}(\sigma) = 5.55$

EXERCISE:-15.3

Question 1:

From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Firstly, the standard deviation of group A is calculated as follows.

Marks	Group A f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
10-20	9	15	–3	9	–27	81
20-30	17	25	–2	4	–34	68

30-40	32	35	–1	1	–32	32
40-50	33	45	0	0	0	0
50-60	40	55	1	1	40	40
60-70	10	65	2	4	20	40
70-80	9	75	3	9	27	81
	150				–6	342

Here, $h = 10$, N = 150, A = 45

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} x_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} = 45 - 0.4 = 44.6$$

$$\sigma_1^2 = \frac{h^2}{N^2}\left(N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right)$$

$$= \frac{100}{22500}\left(150\times 342 - (-6)^2\right)$$

$$= \frac{1}{225}(51264)$$

$$= 227.84$$

$$\therefore \text{Standard deviation}(\sigma_1) = \sqrt{227.84} = 15.09$$

The standard deviation of group B is calculated as follows.

Marks	Group B f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
10-20	10	15	–3	9	–30	90
20-30	20	25	–2	4	–40	80
30-40	30	35	–1	1	–30	30
40-50	25	45	0	0	0	0

50-60	43	55	1	1	43	43
60-70	15	65	2	4	30	60
70-80	7	75	3	9	21	63
	150				–6	366

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} = 45 - 0.4 = 44.6$$

$$\sigma_2^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 366 - (-6)^2\right]$$

$$= \frac{1}{225}\left[54864\right] = 243.84$$

$$\therefore \text{Standard deviation}\,(\sigma_2) = \sqrt{243.84} = 15.61$$

Since the mean of both the groups is same, the group with greater standard deviation will be more variable.

Thus, group B has more variability in the marks.

Question 2:

From the prices of shares X and Y below, find out which is more stable in value:

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

The prices of the shares X are

35, 54, 52, 53, 56, 58, 52, 50, 51, 49

Here, the number of observations, N = 10

$\therefore$ Mean, $\bar{x} = \frac{1}{N}\sum_{i=1}^{10} x_i = \frac{1}{10} \times 510 = 51$

The following table is obtained corresponding to shares X.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
35	–16	256
54	3	9
52	1	1
53	2	4
56	5	25
58	7	49
52	1	1
50	–1	1
51	0	0
49	–2	4
		350

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{10}(xi - \bar{x})^2 = \frac{1}{10} \times 350 = 35$$

$$\therefore \text{Standard deviation}\left(\sigma_1\right) = \sqrt{35} = 5.91$$

$$\text{C.V.}\left(\text{Shares X}\right) = \frac{\sigma_1}{\bar{x}} \times 100 = \frac{5.91}{51} \times 100 = 11.58$$

The prices of share Y are

108, 107, 105, 105, 106, 107, 104, 103, 104, 101

$\therefore$ Mean, $\bar{y} = \frac{1}{N}\sum_{i=1}^{10} y_i = \frac{1}{10} \times 1050 = 105$

The following table is obtained corresponding to shares Y.

y_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	–1	1
103	–2	4
104	–1	1
101	–4	16
		40

$$\text{Variance}\left(\sigma_2^2\right) = \frac{1}{N}\sum_{i=1}^{10}(y_i - \bar{y})^2 = \frac{1}{10} \times 40 = 4$$

$$\therefore \text{Standard deviation}\left(\sigma_2\right) = \sqrt{4} = 2$$

$$\therefore \text{C.V.}\left(\text{Shares Y}\right) = \frac{\sigma_2}{\bar{y}} \times 100 = \frac{2}{105} \times 100 = 1.9 = 11.58$$

C.V. of prices of shares X is greater than the C.V. of prices of shares Y.

Thus, the prices of shares Y are more stable than the prices of shares X.

Question 3:

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	Rs 5253	Rs 5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?

(ii) Which firm, A or B, shows greater variability in individual wages?

(i) Monthly wages of firm A = Rs 5253

Number of wage earners in firm A = 586

∴Total amount paid = Rs 5253 × 586

Monthly wages of firm B = Rs 5253

Number of wage earners in firm B = 648

∴Total amount paid = Rs 5253 × 648

Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

(ii) Variance of the distribution of wages in firm A $\left(\sigma_1^2\right)$ = 100

∴ Standard deviation of the distribution of wages in firm

A ((σ_1) = $\sqrt{100} = 10$

Variance of the distribution of wages in firm $B\left(\sigma_2^2\right)$ = 121

∴ Standard deviation of the distribution of wages in firm $B\left(\sigma_2^2\right) = \sqrt{121} = 11$

The mean of monthly wages of both the firms is same i.e., 5253. Therefore, the firm with greater standard deviation will have more variability.

Thus, firm B has greater variability in the individual wages.

Question 4:

The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

The mean and the standard deviation of goals scored by team A are calculated as follows.

No. of goals scored	**No. of matches**	f_ix_i	x_i^2	$f_ix_i^2$
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
	25	50		130

$$\text{Mean} = \frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{5} f_i} = \frac{50}{25} = 2$$

Thus, the mean of both the teams is same.

$$\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

$$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$$

$$= \frac{1}{25}\sqrt{750}$$

$$= \frac{1}{25} \times 27.38$$

$$= 1.09$$

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2. Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.

Question 5:

The sum and sum of squares corresponding to length *x* (in cm) and weight *y*

(in gm) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \quad \sum_{i=1}^{50} x_i^2 = 902.8, \quad \sum_{i=1}^{50} y_i = 261, \quad \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying, the length or weight?

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8$$

Here, N = 50

∴ Mean, $\overline{x} = \frac{\sum_{i=1}^{50} y_i}{N} = \frac{212}{50} = 4.24$

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{50}(x_i - \bar{x})^2$$
$$= \frac{1}{50}\sum_{i=1}^{50}(x_i - 4.24)^2$$
$$= \frac{1}{50}\sum_{i=1}^{50}\left[x_i^{\ 2} - 8.48x_i + 17.97\right]$$
$$= \frac{1}{50}\left[\sum_{i=1}^{50}x_i^{\ 2} - 8.48\sum_{i=1}^{50}x_i + 17.97\times 50\right]$$

$$= \frac{1}{50}\left[902.8 - 8.48\times(212) + 898.5\right]$$
$$= \frac{1}{50}\left[1801.3 - 1797.76\right]$$
$$= \frac{1}{50}\times 3.54$$
$$= 0.07$$

$$\therefore \text{Standard deviation, } \sigma_1(\text{Length}) = \sqrt{0.07} = 0.26$$
$$\therefore \text{C.V.}(\text{Length}) = \frac{\text{Standard deviation}}{\text{Mean}}\times 100 = \frac{0.26}{4.24}\times 100 = 6.13$$

$$\sum_{i=1}^{50}y_i = 261,\ \sum_{i=1}^{50}y_i^2 = 1457.6$$

Mean, $\bar{y} = \frac{1}{N}\sum_{i=1}^{50}y_i = \frac{1}{50}\times 261 = 5.22$

$$\text{Variance}\left(\sigma_2^2\right)=\frac{1}{N}\sum_{i=1}^{50}(y_i-\bar{y})^2$$

$$=\frac{1}{50}\sum_{i=1}^{50}\left(y_i-5.22\right)^2$$

$$=\frac{1}{50}\sum_{i=1}^{50}\left[y_i^2-10.44y_i+27.24\right]$$

$$=\frac{1}{50}\left[\sum_{i=1}^{50}y_i^2-10.44\sum_{i=1}^{50}y_i+27.24\times 50\right]$$

$$=\frac{1}{50}\left[1457.6-10.44\times(261)+1362\right]$$

$$=\frac{1}{50}\left[2819.6-2724.84\right]$$

$$=\frac{1}{50}\times 94.76$$

$$=1.89$$

$$\therefore \text{Standard deviation}, \sigma_2\left(\text{Weight}\right)=\sqrt{1.89}=1.37$$

$$\therefore \text{C.V.}\left(\text{Weight}\right)=\frac{\text{Standard deviation}}{\text{Mean}}\times 100=\frac{1.37}{5.22}\times 100=26.24$$

Thus, C.V. of weights is greater than the C.V. of lengths. Therefore, weights vary more than the lengths.