**EXERCISE- 3.1**

**Question 1:**

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) – 47° 30' (iii) 240° (iv) 520°

(i) 25°

We know that 180° = π radian

$$\therefore 25^\circ = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) –47° 30'

–47° 30' = $-47\frac{1}{2}$ degree [1° = 60']

$= \frac{-95}{2}$ degree

Since 180° = π radian

$$\frac{-95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right) \pi \text{ radian} = \frac{-19}{72} \pi \text{ radian}$$

$$\therefore -47^\circ\ 30' = \frac{-19}{72} \pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240^\circ = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3} \pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

**Question 2:**

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) – 4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\begin{aligned}\therefore \frac{11}{16} \text{ radain} &= \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree} \\ &= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree} \\ &= 39\frac{3}{8} \text{ deg ree} \\ &= 39° + \frac{3 \times 60}{8} \text{ min utes} \qquad [1° = 60'] \\ &= 39° + 22' + \frac{1}{2} \text{ min utes} \\ &= 39°22'30'' \qquad [1' = 60'']\end{aligned}$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree} = \frac{180 \times 7(-4)}{22} \text{ degree}$$
$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$
$$= -229° + \frac{1 \times 60}{11} \text{ minutes} \qquad [1° = 60']$$
$$= -229° + 5' + \frac{5}{11} \text{ minutes}$$
$$= -229°5'27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ degree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

**Question 3:**

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$
$$= \frac{126}{10} \text{ deg ree} = 12\frac{3}{5} \text{ deg ree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36′.

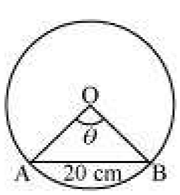
Question 5:

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Diameter of the circle = 40 cm

∴Radius (*r*) of the circle = $\frac{40}{2}$ cm = 20 cm

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

$\therefore \theta = 60° = \frac{\pi}{3}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$.

$$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm.

**Question 6:**

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Let the radii of the two circles be $r_1$ and $r_2$. Let an arc of length $l$ subtend an angle of 60° at the centre of the circle of radius $r_1$, while let an arc of length $l$ subtend an angle of 75° at the centre of the circle of radius $r_2$.

Now, 60° = $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius $r$ unit, if an arc of length $l$ unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$.

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

**Question 7:**

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle $\theta$ radian at the centre, then $\theta = \frac{l}{r}$.

It is given that $r = 75$ cm

(i) Here, $l = 10$ cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, $l = 15$ cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, $l = 21$ cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$

**EXERCISE- 3.2**

Question 1:

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}$, $x$ lies in third quadrant.

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$
$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$
$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since $x$ lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Question 2:

Find the values of other five trigonometric functions if $\sin x = \frac{3}{5}$, $x$ lies in second quadrant.

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$
$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$
$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$
$$\Rightarrow \cos^2 x = \frac{16}{25}$$
$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since $x$ lies in the 2nd quadrant, the value of $\cos x$ will be negative

$$\therefore \cos x = -\frac{4}{5}$$
$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$
$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$
$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

**Question 3:**

Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, $x$ lies in third quadrant.

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since $x$ lies in the 3rd quadrant, the value of sec $x$ will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, $x$ lies in fourth quadrant.

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$
$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$
$$\Rightarrow \sin x = \pm\frac{12}{13}$$

Since $x$ lies in the 4th quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

**Question 5:**

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}$, $x$ lies in second quadrant.

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

Since $x$ lies in the 2nd quadrant, the value of sec $x$ will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

**Question 6:**

Find the value of the trigonometric function sin 765°

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin\left(2\times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}$$

**Question 7:**

Find the value of the trigonometric function cosec (–1410°)

It is known that the values of cosec *x* repeat after an interval of 2π or 360°.

$$\begin{aligned}\therefore \operatorname{cosec}\left(-1410°\right) &= \operatorname{cosec}\left(-1410° + 4\times 360°\right)\\ &= \operatorname{cosec}\left(-1410° + 1440°\right)\\ &= \operatorname{cosec}30° = 2\end{aligned}$$

**Question 8:**

Find the value of the trigonometric function $\tan\frac{19\pi}{3}$

It is known that the values of tan *x* repeat after an interval of π or 180°.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$$

**Question 9:**

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

It is known that the values of sin *x* repeat after an interval of 2π or 360°.

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

**Question 10:**

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

It is known that the values of cot $x$ repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)=\cot\frac{\pi}{4}=1$$

**EXERCISE- 3.3**

**Question 1:**

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

L.H.S. = $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$
$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$
$$=\text{R.H.S.}$$

**Question 2:**

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

L.H.S. = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$= 2 \times \frac{1}{4} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$$

$$= \text{R.H.S.}$$

**Question 3:**

Prove that $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

L.H.S. = $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$$= \left(\sqrt{3}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \operatorname{cosec}\frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

$$= \text{R.H.S}$$

**Question 4:**

Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

L.H.S = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$
$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2\times\frac{1}{2} + 8$$
$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$
$$= 1 + 1 + 8$$
$$= 10$$
$$= \text{R.H.S}$$

**Question 5:**

Find the value of:

(i) sin 75°

(ii) tan 15°

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[$\sin(x + y) = \sin x \cos y + \cos x \sin y$]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3 + 1 - 2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$
$$= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}$$

**Question 6:**

Prove that: $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$\left[\begin{array}{l}\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$

$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]$$

$$=\sin(x+y)$$

$$=\text{R.H.S}$$

**Question 7:**

Prove that: $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

∴L.H.S. =

Question 8:

Prove that $$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$
$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$
$$=\frac{-\cos^2 x}{-\sin^2 x}$$
$$=\cot^2 x$$
$$=\text{R.H.S.}$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

L.H.S. = $$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$=\sin x\cos x[\tan x+\cot x]$$
$$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$
$$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$
$$=1=\text{R.H.S.}$$

Question 10:

Prove that $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

L.H.S. = sin $(n + 1)x$ sin$(n + 2)x$ + cos $(n + 1)x$ cos$(n + 2)x$

$$=\frac{1}{2}\left[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x\right]$$

$$=\frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$$

$$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$$

$$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$=\cos(-x)=\cos x=\text{R.H.S.}$$

**Question 11:**

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

It is known that $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$.

$\therefore$L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$$=\text{R.H.S.}$$

**Question 12:**

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore$L.H.S. $= \sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

$= (2 \sin 5x \cos x)(2 \cos 5x \sin x)$

$= (2 \sin 5x \cos 5x)(2 \sin x \cos x)$

$= \sin 10x \sin 2x$

= R.H.S.

**Question 13:**

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

It is known that

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$L.H.S. $= \cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - 6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

$= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$

$= (2 \sin 4x \cos 4x)(2 \sin 2x \cos 2x)$

$= \sin 8x \sin 4x$

= R.H.S.

**Question 14:**

Prove that $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

L.H.S. $= \sin 2x + 2 \sin 4x + \sin 6x$

$= [\sin 2x + \sin 6x] + 2 \sin 4x$

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$= 2 \sin 4x \cos (-2x) + 2 \sin 4x$

$= 2 \sin 4x \cos 2x + 2 \sin 4x$

$= 2 \sin 4x (\cos 2x + 1)$

$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$

$= 2 \sin 4x (2 \cos^2 x)$

$= 4\cos^2 x \sin 4x$

= R.H.S.

**Question 15:**

Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

L.H.S $= \cot 4x (\sin 5x + \sin 3x)$

$$=\frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

= 2 cos 4$x$ cos $x$

R.H.S. = cot $x$ (sin 5$x$ – sin 3$x$)

$$=\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

= 2 cos 4$x$. cos $x$

L.H.S. = R.H.S.

**Question 16:**

Prove that $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S}=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$$=\text{R.H.S.}$$

**Question 17:**

Prove that $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$=\frac{\sin 4x}{\cos 4x}$$

$$=\tan 4x=\text{R.H.S.}$$

**Question 18:**

Prove that $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\frac{x-y}{2}$

It is known that

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$, $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore$L.H.S. $= \frac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

**Question 19:**

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore$L.H.S. $= \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

**Question 20:**

Prove that $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \cos^2 A-\sin^2 A=\cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

$$=\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$

$$=-2\times(-\sin x)$$

$$=2\sin x=\text{R.H.S.}$$

**Question 21:**

Prove that $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

$$\text{L.H.S.} = \frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$=\cot 3x=\text{R.H.S.}$$

### Question 22:

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

L.H.S. $= \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$

$= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

$$\left[\because \cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$

$= \cot x \cot 2x - (\cot 2x \cot x - 1)$

$= 1 =$ R.H.S.

### Question 23:

Prove that $\tan 4x=\dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

It is known that $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

$\therefore$L.H.S. = tan 4x = tan 2(2x)

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[\dfrac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

**Question 24:**

Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

L.H.S. = $\cos 4x$

= $\cos 2(2x)$

= $1 - 2\sin^2 2x$ [$\cos 2A = 1 - 2\sin^2 A$]

$= 1 - 2(2 \sin x \cos x)^2$ [$\sin 2A = 2\sin A \cos A$]

$= 1 - 8 \sin^2 x \cos^2 x$

= R.H.S.

**Question 25:**

Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

L.H.S. $= \cos 6x$

$= \cos 3(2x)$

$= 4 \cos^3 2x - 3 \cos 2x$ [$\cos 3A = 4 \cos^3 A - 3 \cos A$]

$= 4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)$ [$\cos 2x = 2 \cos^2 x - 1$]

$= 4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6\cos^2 x + 3$

$= 4 [8\cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$

$= 32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$

$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

= R.H.S.

## EXERCISE- 3.4

**Question 1:**

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

$\tan x = \sqrt{3}$

It is known that $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

$$\text{Now, } \tan x = \tan\frac{\pi}{3}$$
$$\Rightarrow x = n\pi + \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Question 2:

Find the principal and general solutions of the equation $\sec x = 2$

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

$$\text{Now, } \sec x = \sec\frac{\pi}{3}$$
$$\Rightarrow \cos x = \cos\frac{\pi}{3} \qquad \left[\sec x = \frac{1}{\cos x}\right]$$
$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

Question 3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

$\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6}=\sqrt{3}$

$\therefore \cot\left(\pi-\frac{\pi}{6}\right)=-\cot\frac{\pi}{6}=-\sqrt{3}$ and $\cot\left(2\pi-\frac{\pi}{6}\right)=-\cot\frac{\pi}{6}=-\sqrt{3}$

i.e., $\cot\frac{5\pi}{6}=-\sqrt{3}$ and $\cot\frac{11\pi}{6}=-\sqrt{3}$

Therefore, the principal solutions are $x=\frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x=\cot\frac{5\pi}{6}$

$\Rightarrow \tan x=\tan\frac{5\pi}{6}$ $\quad\left[\cot x=\frac{1}{\tan x}\right]$

$\Rightarrow x=n\pi+\frac{5\pi}{6}$, where $n\in Z$

Therefore, the general solution is $x=n\pi+\frac{5\pi}{6}$, where $n\in Z$

**Question 4:**

Find the general solution of cosec $x$ = –2

cosec $x$ = –2

It is known that

$\operatorname{cosec}\frac{\pi}{6}=2$

$\therefore \operatorname{cosec}\left(\pi+\frac{\pi}{6}\right)=-\operatorname{cosec}\frac{\pi}{6}=-2$ and $\operatorname{cosec}\left(2\pi-\frac{\pi}{6}\right)=-\operatorname{cosec}\frac{\pi}{6}=-2$

i.e., $\operatorname{cosec}\frac{7\pi}{6}=-2$ and $\operatorname{cosec}\frac{11\pi}{6}=-2$

Therefore, the principal solutions are $x=\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$

**Question 5:**

Find the general solution of the equation $\cos 4x = \cos 2x$

$$\cos 4x = \cos 2x$$

$$\Rightarrow \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$

$$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$

$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$

**Question 6:**

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

$$\cos 3x + \cos x - \cos 2x = 0$$

$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-\cos 2x=0 \quad \left[\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\cos 2x\cos x-\cos 2x=0$$

$$\Rightarrow \cos 2x\left(2\cos x-1\right)=0$$

$$\Rightarrow \cos 2x=0 \qquad \text{or} \qquad 2\cos x-1=0$$

$$\Rightarrow \cos 2x=0 \qquad \text{or} \qquad \cos x=\frac{1}{2}$$

$$\therefore 2x=\left(2n+1\right)\frac{\pi}{2} \qquad \text{or} \qquad \cos x=\cos\frac{\pi}{3},\ \text{where } n\in Z$$

$$\Rightarrow x=\left(2n+1\right)\frac{\pi}{4} \qquad \text{or} \qquad x=2n\pi\pm\frac{\pi}{3},\ \text{where } n\in Z$$

**Question 7:**

Find the general solution of the equation $\sin 2x+\cos x=0$

$$\sin 2x+\cos x=0$$

$$\Rightarrow 2\sin x\cos x+\cos x=0$$

$$\Rightarrow \cos x\left(2\sin x+1\right)=0$$

$$\Rightarrow \cos x=0 \qquad \text{or} \qquad 2\sin x+1=0$$

$$\text{Now, } \cos x=0\Rightarrow \cos x=\left(2n+1\right)\frac{\pi}{2},\ \text{where } n\in Z$$

$$2\sin x+1=0$$

$$\Rightarrow \sin x=\frac{-1}{2}=-\sin\frac{\pi}{6}=\sin\left(\pi+\frac{\pi}{6}\right)=\sin\left(\pi+\frac{\pi}{6}\right)=\sin\frac{7\pi}{6}$$

$$\Rightarrow x=n\pi+\left(-1\right)^n\frac{7\pi}{6},\ \text{where } n\in Z$$

Therefore, the general solution is $\left(2n+1\right)\frac{\pi}{2}$ or $n\pi+\left(-1\right)^n\frac{7\pi}{6},\ n\in Z$.

**Question 8:**

Find the general solution of the equation $\sec^2 2x=1-\tan 2x$

$$\sec^2 2x=1-\tan 2x$$

$\Rightarrow 1+\tan^2 2x = 1-\tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0 \quad$ or $\quad \tan 2x + 1 = 0$

Now, $\tan 2x = 0$

$\Rightarrow \tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, where $n \in Z$

$\Rightarrow x = \dfrac{n\pi}{2}$, where $n \in Z$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\dfrac{\pi}{4} = \tan\left(\pi - \dfrac{\pi}{4}\right) = \tan\dfrac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \dfrac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\dfrac{n\pi}{2}$ or $\dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, $n \in Z$.

Question 9:

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

$\sin x + \sin 3x + \sin 5x = 0$

$(\sin x + \sin 5x) + \sin 3x = 0$

$\Rightarrow \left[2\sin\left(\dfrac{x+5x}{2}\right)\cos\left(\dfrac{x-5x}{2}\right)\right] + \sin 3x = 0 \qquad \left[\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x \cos(-2x) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x+1)=0$

$\Rightarrow \sin 3x=0 \quad \text{or} \quad 2\cos 2x+1=0$

Now, $\sin 3x=0 \Rightarrow 3x=n\pi$, where $n \in Z$

i.e., $x=\dfrac{n\pi}{3}$, where $n \in Z$

$2\cos 2x+1=0$

$\Rightarrow \cos 2x=\dfrac{-1}{2}=-\cos\dfrac{\pi}{3}=\cos\left(\pi-\dfrac{\pi}{3}\right)$

$\Rightarrow \cos 2x=\cos\dfrac{2\pi}{3}$

$\Rightarrow 2x=2n\pi\pm\dfrac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x=n\pi\pm\dfrac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $\dfrac{n\pi}{3}$ or $n\pi\pm\dfrac{\pi}{3}$, $n \in Z$