Exercise – 10.1

Question 1:

How many tangents can a circle have?

A circle can have infinite tangents.

Question 2:

Fill in the blanks:

(i) A tangent to a circle intersects it in _______ point (s).

(ii) A line intersecting a circle in two points is called a __________.

(iii) A circle can have __________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ____.

(i) One

(ii) Secant

(iii) Two

(iv) Point of contact

Question 3:

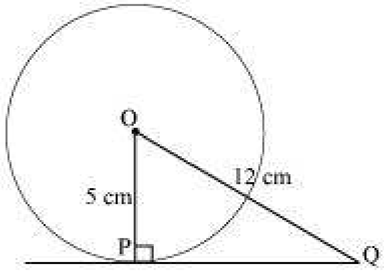
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm. (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

We know that the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

$\therefore OP \perp PQ$

By applying Pythagoras theorem in ΔOPQ,



$\therefore OP^2 + PQ^2 = OQ^2$

$5^2 + PQ^2 = 12^2$

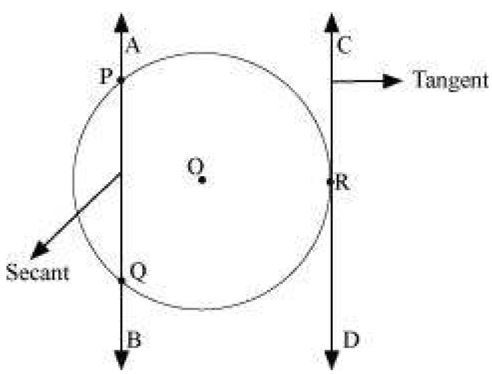
$PQ^2 = 144 - 25$

$PQ = \sqrt{119}$ cm.

Hence, the correct answer is (D).

Question 4:

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.



It can be observed that AB and CD are two parallel lines. Line AB is intersecting the circle at exactly two points, P and Q. Therefore, line AB is the secant of this circle. Since line CD is intersecting the circle at exactly one point, R, line CD is the tangent to the circle.

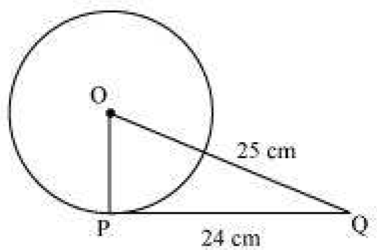
Exercise – 10.2

Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP ⊥ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

OP = 7

Therefore, the radius of the circle is 7 cm.

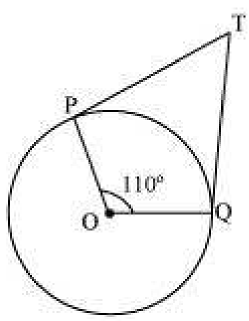
Hence, alternative (A) is correct.

Question 2:

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = 90^\circ$

$\angle OQT = 90^\circ$

In quadrilateral POQT,

Sum of all interior angles $= 360^\circ$

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 70°$

Hence, alternative (B) is correct.

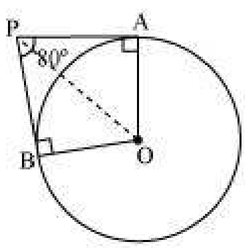
Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp PA$ and $OB \perp PB$

$\angle OBP = 90°$

$\angle OAP = 90°$

In AOBP,

Sum of all interior angles = 360°

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$90° + 80° + 90° + \angle BOA = 360°$

$\angle BOA = 100°$

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

A ↔ B, P ↔ P, O ↔ O

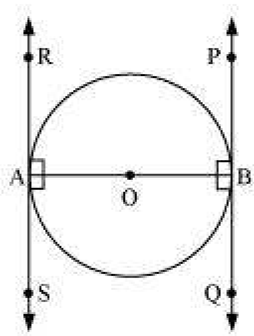
And thus, ∠POB = ∠POA

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Question 4:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ RS and OB ⊥ PQ

∠OAR = 90º

∠OAS = 90º

∠OBP = 90º

∠OBQ = 90º

It can be observed that

∠OAR = ∠OBQ (Alternate interior angles)

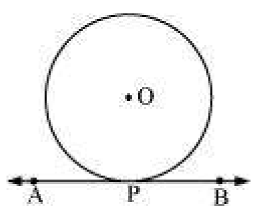
∠OAS = ∠OBP (Alternate interior angles)

Since alternate interior angles are equal, lines PQ and RS will be parallel.

Question 5:

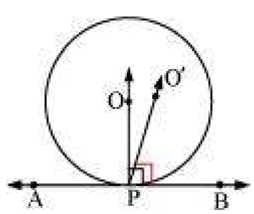
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$\angle O'PB = 90°$ … (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore \angle OPB = 90°$ … (2)

Comparing equations (1) and (2), we obtain

$\angle O'PB = \angle OPB$ … (3)

From the figure, it can be observed that,

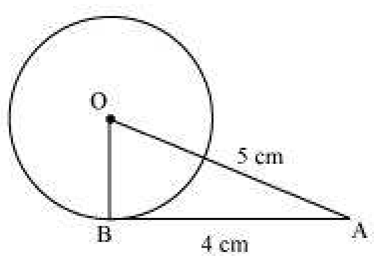
$\angle O'PB < \angle OPB$ … (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.

Question 6:

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

OB ⊥ AB (Radius ⊥ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

$16 + BO^2 = 25$

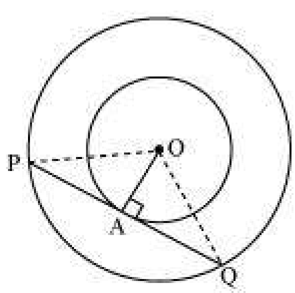
$BO^2 = 9$

BO = 3

Hence, the radius of the circle is 3 cm.

Question 7:

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

OA ⊥ PQ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$9 + AP^2 = 25$

$AP^2 = 16$

$AP = 4$

In ΔOPQ,

Since OA ⊥ PQ,

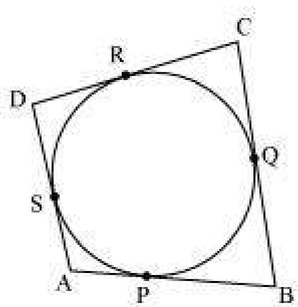
AP = AQ (Perpendicular from the center of the circle bisects the chord)

$\therefore PQ = 2AP = 2 \times 4 = 8$

Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



It can be observed that

DR = DS (Tangents on the circle from point D) … (1)

CR = CQ (Tangents on the circle from point C) … (2)

BP = BQ (Tangents on the circle from point B) … (3)

AP = AS (Tangents on the circle from point A) … (4)

Adding all these equations, we obtain

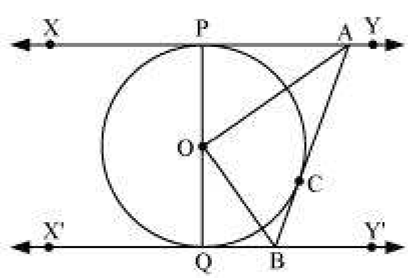
DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

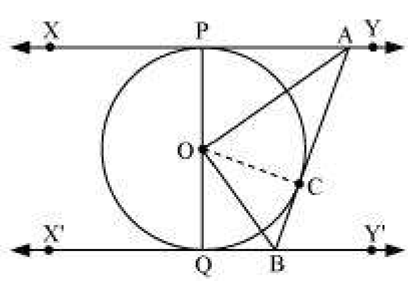
CD + AB = AD + BC

Question 9:

In the given figure, XY and X’Y’ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X’Y’ at B. Prove that ∠AOB=90°.



Let us join point O to C.



In ΔOPA and ΔOCA,

OP = OC (Radii of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

$\Delta OPA \cong \Delta OCA$ (SSS congruence criterion)

Therefore, $P \leftrightarrow C$, $A \leftrightarrow A$, $O \leftrightarrow O$

$\angle POA = \angle COA$ … (*i*)

Similarly, $\Delta OQB \cong \Delta OCB$

$\angle QOB = \angle COB$ … (*ii*)

Since POQ is a diameter of the circle, it is a straight line.

Therefore, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

From equations (*i*) and (*ii*), it can be observed that

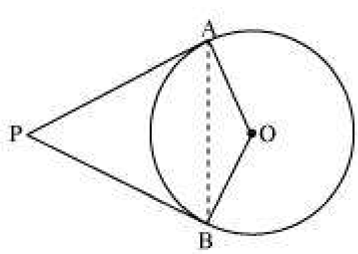
$2\angle COA + 2\angle COB = 180^\circ$

$\angle COA + \angle COB = 90^\circ$

$\angle AOB = 90^\circ$

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of the circle.

It can be observed that

OA (radius) $\perp$ PA (tangent)

Therefore, $\angle OAP = 90°$

Similarly, OB (radius) $\perp$ PB (tangent)

$\angle OBP = 90°$

In quadrilateral OAPB,

Sum of all interior angles = 360º

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360º$

$90º + \angle APB + 90º + \angle BOA = 360º$

$\angle APB + \angle BOA = 180º$

Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

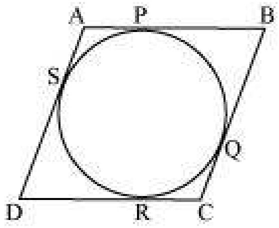
Question 11:

Prove that the parallelogram circumscribing a circle is a rhombus.

Since ABCD is a parallelogram,

AB = CD …(1)

BC = AD …(2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

AB = BC …(3)

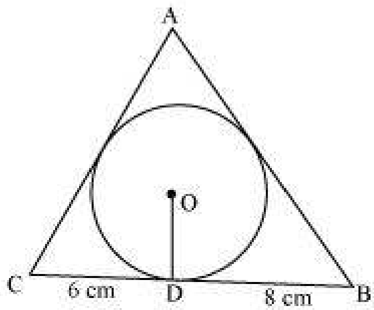
Comparing equations (1), (2), and (3), we obtain

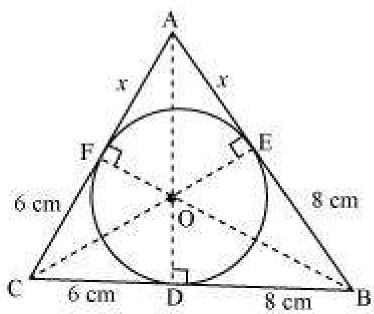
AB = BC = CD = DA

Hence, ABCD is a rhombus.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.





Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be *x*.

In ΔABC,

CF = CD = 6cm (Tangents on the circle from point C)

BE = BD = 8cm (Tangents on the circle from point B)

AE = AF = x (Tangents on the circle from point A)

AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

$2s$ = AB + BC + CA

$= x + 8 + 14 + 6 + x$

$= 28 + 2x$

$s = 14 + x$

$$\begin{aligned}\text{Area of } \Delta\text{ABC} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}\\ &= \sqrt{(14+x)(x)(8)(6)}\\ &= 4\sqrt{3(14x+x^2)}\end{aligned}$$

Area of ΔOBC = $\frac{1}{2} \times \text{OD} \times \text{BC} = \frac{1}{2} \times 4 \times 14 = 28$

Area of ΔOCA = $\frac{1}{2} \times \text{OF} \times \text{AC} = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$

Area of ΔOAB = $\frac{1}{2} \times \text{OE} \times \text{AB} = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$4\sqrt{3\left(14x+x^2\right)} = 28 + 12 + 2x + 16 + 2x$

$\Rightarrow 4\sqrt{3\left(14x+x^2\right)} = 56+4x$

$\Rightarrow \sqrt{3\left(14x+x^2\right)} = 14+x$

$\Rightarrow 3\left(14x+x^2\right) = \left(14+x\right)^2$

$\Rightarrow 42x+3x^2 = 196+x^2+28x$

$\Rightarrow 2x^2+14x-196 = 0$

$\Rightarrow x^2+7x-98=0$

$\Rightarrow x^2+14x-7x-98=0$

$\Rightarrow x\left(x+14\right)-7\left(x+14\right)=0$

$\Rightarrow \left(x+14\right)\left(x-7\right)=0$

Either $x+14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7

However, $x = -14$ is not possible as the length of the sides will be negative.

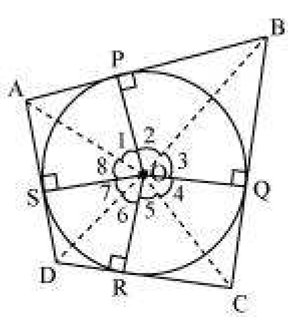
Therefore, $x = 7$

Hence, AB $= x + 8 = 7 + 8 = 15$ cm

CA $= 6 + x = 6 + 7 = 13$ cm

Question 13:

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

$\Delta OAP \cong \Delta OAS$ (SSS congruence criterion)

Therefore, $A \leftrightarrow A$, $P \leftrightarrow S$, $O \leftrightarrow O$

And thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^o$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^o$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^o$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^o$

$\angle AOB + \angle COD = 180^o$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^o$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.