



Exercise – 9.1

Question 1:

Determine order and degree(if defined) of differential equation $\frac{d^4 y}{dx^4} + \sin(y''') = 0$

$$\begin{aligned}\frac{d^4 y}{dx^4} + \sin(y''') &= 0 \\ \Rightarrow y'''' + \sin(y''') &= 0\end{aligned}$$

The highest order derivative present in the differential equation is y'''' . Therefore, its order is four.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

Question 2:

Determine order and degree(if defined) of differential equation $y' + 5y = 0$

The given differential equation is:

$$y' + 5y = 0$$

The highest order derivative present in the differential equation is y' . Therefore, its order is one.

It is a polynomial equation in y' . The highest power raised to y' is 1. Hence, its degree is one.

Question 3:

Determine order and degree(if defined) of differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2 s}{dt^2} = 0$

$$\left(\frac{ds}{dt}\right)^4 + 3\frac{d^2 s}{dt^2} = 0$$



The highest order derivative present in the given differential equation is $\frac{d^2s}{dt^2}$. Therefore, its order is two.

It is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$. The power raised to $\frac{d^2s}{dt^2}$ is 1.

Hence, its degree is one.

Question 4:

Determine order and degree(if defined) of differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is 2.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

Question 5:

Determine order and degree(if defined) of differential equation $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} - \cos 3x - \sin 3x = 0$$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.



It is a polynomial equation in $\frac{d^2y}{dx^2}$ and the power raised to $\frac{d^2y}{dx^2}$ is 1.

Hence, its degree is one.

Question 6:

Determine order and degree(if defined) of differential

equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

$$(y''')^2 + (y'')^3 + (y') + y^5 = 0$$

The highest order derivative present in the differential equation is y''' . Therefore, its order is three.

The given differential equation is a polynomial equation in $y''', y'',$ and y' .

The highest power raised to y''' is 2. Hence, its degree is 2.

Question 7:

Determine order and degree(if defined) of differential equation $y''' + 2y'' + y' = 0$

$$y''' + 2y'' + y' = 0$$

The highest order derivative present in the differential equation is y''' . Therefore, its order is three.

It is a polynomial equation in y''', y'' and y' . The highest power raised to y''' is 1. Hence, its degree is 1.

Question 8:

Determine order and degree(if defined) of differential equation $y' + y = e^x$

$$y' + y = e^x$$
$$\Rightarrow y' + y - e^x = 0$$



The highest order derivative present in the differential equation is y' . Therefore, its order is one.

The given differential equation is a polynomial equation in y' and the highest power raised to y' is one. Hence, its degree is one.

Question 9:

Determine order and degree(if defined) of differential equation $y'' + (y')^2 + 2y = 0$

$$y'' + (y')^2 + 2y = 0$$

The highest order derivative present in the differential equation is y'' . Therefore, its order is two.

The given differential equation is a polynomial equation in y'' and y' and the highest power raised to y'' is one.

Hence, its degree is one.

Question 10:

Determine order and degree(if defined) of differential equation $y'' + 2y' + \sin y = 0$

$$y'' + 2y' + \sin y = 0$$

The highest order derivative present in the differential equation is y'' . Therefore, its order is two.

This is a polynomial equation in y'' and y' and the highest power raised to y'' is one. Hence, its degree is one.

Question 11:

The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$
 is



(A) 3 (B) 2 (C) 1 (D) not defined

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

The given differential equation is not a polynomial equation in its derivatives. Therefore, its degree is not defined.

Hence, the correct answer is D.

Question 12:

The order of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0 \text{ is}$$

(A) 2 (B) 1 (C) 0 (D) not defined

$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

Hence, the correct answer is A.

Exercise – 9.2

Question 1:

$$y = e^x + 1 \quad : \quad y'' - y' = 0$$

$$y = e^x + 1$$

Differentiating both sides of this equation with respect to x , we get:

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(e^x + 1) \\ \Rightarrow y' &= e^x \end{aligned} \quad \dots(1)$$



Now, differentiating equation (1) with respect to x , we get:

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$
$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S. as:

$$y'' - y' = e^x - e^x = 0 = \text{R.H.S.}$$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$$y = x^2 + 2x + C \quad : \quad y' - 2x - 2 = 0$$

$$y = x^2 + 2x + C$$

Differentiating both sides of this equation with respect to x , we get:

$$y' = \frac{d}{dx}(x^2 + 2x + C)$$
$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = y' - 2x - 2 = 2x + 2 - 2x - 2 = 0 = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 3:

$$y = \cos x + C \quad : \quad y' + \sin x = 0$$

$$y = \cos x + C$$

Differentiating both sides of this equation with respect to x , we get:



$$y' = \frac{d}{dx}(\cos x + C)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = y' + \sin x = -\sin x + \sin x = 0 = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 4:

$$y = \sqrt{1+x^2} \quad : \quad y' = \frac{xy}{1+x^2}$$

$$y = \sqrt{1+x^2}$$

Differentiating both sides of the equation with respect to x , we get:

$$y' = \frac{d}{dx}(\sqrt{1+x^2})$$

$$y' = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}(1+x^2)$$

$$y' = \frac{2x}{2\sqrt{1+x^2}}$$

$$y' = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 5:



$$y = Ax \quad : \quad xy' = y(x \neq 0)$$
$$y = Ax$$

Differentiating both sides with respect to x , we get:

$$y' = \frac{d}{dx}(Ax)$$
$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = xy' = x \cdot A = Ax = y = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation

Question 6:

$$y = x \sin x \quad : \quad xy' = y + x\sqrt{x^2 - y^2} \quad (x \neq 0 \text{ and } x > y \text{ or } x < -y)$$

$$y = x \sin x$$

Differentiating both sides of this equation with respect to x , we get:

$$y' = \frac{d}{dx}(x \sin x)$$
$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$
$$\Rightarrow y' = \sin x + x \cos x$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned} \text{L.H.S.} &= xy' = x(\sin x + x \cos x) \\ &= x \sin x + x^2 \cos x \\ &= y + x^2 \cdot \sqrt{1 - \sin^2 x} \\ &= y + x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2} \\ &= y + x \sqrt{y^2 - x^2} \\ &= \text{R.H.S.} \end{aligned}$$



Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$$xy = \log y + C \quad : \quad y' = \frac{y^2}{1-xy} \quad (xy \neq 1)$$

$$xy = \log y + C$$

Differentiating both sides of this equation with respect to x , we get:

$$\begin{aligned} \frac{d}{dx}(xy) &= \frac{d}{dx}(\log y) \\ \Rightarrow y \cdot \frac{d}{dx}(x) + x \cdot \frac{dy}{dx} &= \frac{1}{y} \frac{dy}{dx} \\ \Rightarrow y + xy' &= \frac{1}{y} y' \\ \Rightarrow y^2 + xy y' &= y' \\ \Rightarrow (xy - 1) y' &= -y^2 \\ \Rightarrow y' &= \frac{y^2}{1-xy} \end{aligned}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$$y - \cos y = x \quad : \quad (y \sin y + \cos y + x) y' = y$$

$$y - \cos y = x \quad \dots(1)$$

Differentiating both sides of the equation with respect to x , we get:

$$\begin{aligned} \frac{dy}{dx} - \frac{d}{dx}(\cos y) &= \frac{d}{dx}(x) \\ \Rightarrow y' + \sin y \cdot y' &= 1 \\ \Rightarrow y'(1 + \sin y) &= 1 \\ \Rightarrow y' &= \frac{1}{1 + \sin y} \end{aligned}$$



Substituting the value of y' in equation (1), we get:

$$\begin{aligned}\text{L.H.S.} &= (y \sin y + \cos y + x) y' \\ &= (y \sin y + \cos y + y - \cos y) \times \frac{1}{1 + \sin y} \\ &= y(1 + \sin y) \cdot \frac{1}{1 + \sin y} \\ &= y \\ &= \text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

$$x + y = \tan^{-1} y \quad : \quad y^2 y' + y^2 + 1 = 0$$

$$x + y = \tan^{-1} y$$

Differentiating both sides of this equation with respect to x , we get:

$$\begin{aligned}\frac{d}{dx}(x + y) &= \frac{d}{dx}(\tan^{-1} y) \\ \Rightarrow 1 + y' &= \left[\frac{1}{1 + y^2} \right] y' \\ \Rightarrow y' \left[\frac{1}{1 + y^2} - 1 \right] &= 1 \\ \Rightarrow y' \left[\frac{1 - (1 + y^2)}{1 + y^2} \right] &= 1 \\ \Rightarrow y' \left[\frac{-y^2}{1 + y^2} \right] &= 1 \\ \Rightarrow y' &= \frac{-(1 + y^2)}{y^2}\end{aligned}$$

Substituting the value of y' in the given differential equation, we get:



$$\begin{aligned} \text{L.H.S.} &= y^2 y' + y^2 + 1 = y^2 \left[\frac{-(1+y^2)}{y^2} \right] + y^2 + 1 \\ &= -1 - y^2 + y^2 + 1 \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 10:

$$y = \sqrt{a^2 - x^2} \quad x \in (-a, a) \quad : \quad x + y \frac{dy}{dx} = 0 \quad (y \neq 0)$$

$$y = \sqrt{a^2 - x^2}$$

Differentiating both sides of this equation with respect to x , we get:

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\sqrt{a^2 - x^2}) \\ \Rightarrow \frac{dy}{dx} &= \frac{1}{2\sqrt{a^2 - x^2}} \cdot \frac{d}{dx} (a^2 - x^2) \\ &= \frac{1}{2\sqrt{a^2 - x^2}} (-2x) \\ &= \frac{-x}{\sqrt{a^2 - x^2}} \end{aligned}$$

Substituting the value of $\frac{dy}{dx}$ in the given differential equation, we get:

$$\begin{aligned} \text{L.H.S.} &= x + y \frac{dy}{dx} = x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}} \\ &= x - x \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 11:



The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Question 12:

The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

In a particular solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.

Exercise – 9.3

Question 1:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Differentiating both sides of the given equation with respect to x , we get:

$$\frac{1}{a} + \frac{1}{b} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} y' = 0$$

Again, differentiating both sides with respect to x , we get:



$$0 + \frac{1}{b} y'' = 0$$

$$\Rightarrow \frac{1}{b} y'' = 0$$

$$\Rightarrow y'' = 0$$

Hence, the required differential equation of the given curve is $y'' = 0$.

Question 2:

$$y^2 = a(b^2 - x^2)$$

$$y^2 = a(b^2 - x^2)$$

Differentiating both sides with respect to x , we get:

$$2y \frac{dy}{dx} = a(-2x)$$

$$\Rightarrow 2yy' = -2ax$$

$$\Rightarrow yy' = -ax \quad \dots(1)$$

Again, differentiating both sides with respect to x , we get:

$$y' \cdot y' + yy'' = -a$$

$$\Rightarrow (y')^2 + yy'' = -a \quad \dots(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{(y')^2 + yy''}{yy'} = \frac{-a}{-ax}$$

$$\Rightarrow xy y'' + x(y')^2 - yy'' = 0$$

This is the required differential equation of the given curve.

Question 3:

$$y = ae^{3x} + be^{-2x}$$

$$y = ae^{3x} + be^{-2x} \quad \dots(1)$$



Differentiating both sides with respect to x , we get:

$$y' = 3ae^{3x} - 2be^{-2x} \quad \dots(2)$$

Again, differentiating both sides with respect to x , we get:

$$y'' = 9ae^{3x} + 4be^{-2x} \quad \dots(3)$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\begin{aligned} (2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2be^{-2x}) &= 2y + y' \\ \Rightarrow 5ae^{3x} &= 2y + y' \\ \Rightarrow ae^{3x} &= \frac{2y + y'}{5} \end{aligned}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$\begin{aligned} (3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) &= 3y - y' \\ \Rightarrow 5be^{-2x} &= 3y - y' \\ \Rightarrow be^{-2x} &= \frac{3y - y'}{5} \end{aligned}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$\begin{aligned} y'' &= 9 \cdot \frac{(2y + y')}{5} + 4 \cdot \frac{(3y - y')}{5} \\ \Rightarrow y'' &= \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5} \\ \Rightarrow y'' &= \frac{30y + 5y'}{5} \\ \Rightarrow y'' &= 6y + y' \\ \Rightarrow y'' - y' - 6y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Question 4:

$$y = e^{2x}(a + bx)$$



$$y = e^{2x}(a + bx) \quad \dots(1)$$

Differentiating both sides with respect to x , we get:

$$\begin{aligned} y' &= 2e^{2x}(a + bx) + e^{2x} \cdot b \\ \Rightarrow y' &= e^{2x}(2a + 2bx + b) \quad \dots(2) \end{aligned}$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$\begin{aligned} y' - 2y &= e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx) \\ \Rightarrow y' - 2y &= be^{2x} \quad \dots(3) \end{aligned}$$

Differentiating both sides with respect to x , we get:

$$y''k - 2y' = 2be^{2x} \quad \dots(4)$$

Dividing equation (4) by equation (3), we get:

$$\begin{aligned} \frac{y'' - 2y'}{y' - 2y} &= 2 \\ \Rightarrow y'' - 2y' &= 2y' - 4y \\ \Rightarrow y'' - 4y' + 4y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Question 5:

$$y = e^x(a \cos x + b \sin x)$$

$$y = e^x(a \cos x + b \sin x) \quad \dots(1)$$

Differentiating both sides with respect to x , we get:

$$\begin{aligned} y' &= e^x(a \cos x + b \sin x) + e^x(-a \sin x + b \cos x) \\ \Rightarrow y' &= e^x[(a + b) \cos x - (a - b) \sin x] \quad \dots(2) \end{aligned}$$

Again, differentiating with respect to x , we get:



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$$y'' = e^x [(a+b)\cos x - (a-b)\sin x] + e^x [-(a+b)\sin x - (a-b)\cos x]$$

$$y'' = e^x [2b\cos x - 2a\sin x]$$

$$y'' = 2e^x (b\cos x - a\sin x)$$

$$\Rightarrow \frac{y''}{2} = e^x (b\cos x - a\sin x) \quad \dots(3)$$

Adding equations (1) and (3), we get:

$$y + \frac{y''}{2} = e^x [(a+b)\cos x - (a-b)\sin x]$$

$$\Rightarrow y + \frac{y''}{2} = y'$$

$$\Rightarrow 2y + y'' = 2y'$$

$$\Rightarrow y'' - 2y' + 2y = 0$$

This is the required differential equation of the given curve.

Question 6:

Form the differential equation of the family of circles touching the y -axis at the origin.

The centre of the circle touching the y -axis at origin lies on the x -axis.

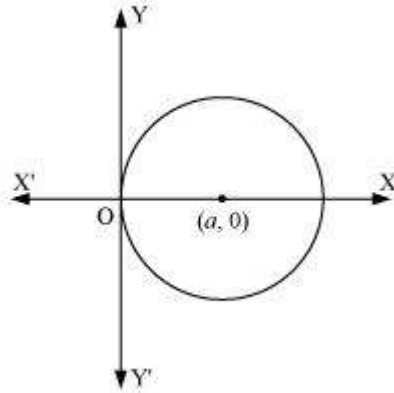
Let $(a, 0)$ be the centre of the circle.

Since it touches the y -axis at origin, its radius is a .

Now, the equation of the circle with centre $(a, 0)$ and radius (a) is

$$(x-a)^2 + y^2 = a^2.$$

$$\Rightarrow x^2 + y^2 = 2ax \quad \dots(1)$$



Differentiating equation (1) with respect to x , we get:

$$2x + 2yy' = 2a$$

$$\Rightarrow x + yy' = a$$

Now, on substituting the value of a in equation (1), we get:

$$x^2 + y^2 = 2(x + yy')x$$

$$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$$

$$\Rightarrow 2xyy' + x^2 = y^2$$

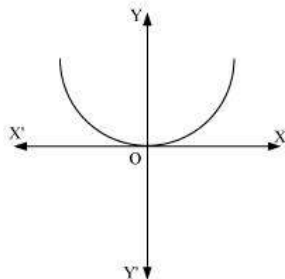
This is the required differential equation.

Question 7:

Form the differential equation of the family of parabolas having vertex at origin and axis along positive y -axis.

The equation of the parabola having the vertex at origin and the axis along the positive y -axis is:

$$x^2 = 4ay \quad \dots(1)$$





Differentiating equation (1) with respect to x , we get:

$$2x = 4ay' \quad \dots(2)$$

Dividing equation (2) by equation (1), we get:

$$\begin{aligned} \frac{2x}{x^2} &= \frac{4ay'}{4ay} \\ \Rightarrow \frac{2}{x} &= \frac{y'}{y} \\ \Rightarrow xy' &= 2y \\ \Rightarrow xy' - 2y &= 0 \end{aligned}$$

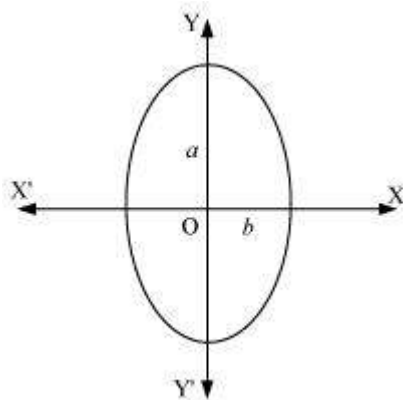
This is the required differential equation.

Question 8:

Form the differential equation of the family of ellipses having foci on y -axis and centre at origin.

The equation of the family of ellipses having foci on the y -axis and the centre at origin is as follows:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \dots(1)$$



Differentiating equation (1) with respect to x , we get:



$$\frac{2x}{b^2} + \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad \dots(2)$$

Again, differentiating with respect to x , we get:

$$\frac{1}{b^2} + \frac{y' \cdot y' + y \cdot y''}{a^2} = 0$$

$$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy'') = 0$$

$$\Rightarrow \frac{1}{b^2} = -\frac{1}{a^2}(y'^2 + yy'')$$

Substituting this value in equation (2), we get:

$$x \left[-\frac{1}{a^2}(y'^2 + yy'') \right] + \frac{yy'}{a^2} = 0$$

$$\Rightarrow -x(y')^2 - xyy'' + yy' = 0$$

$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$

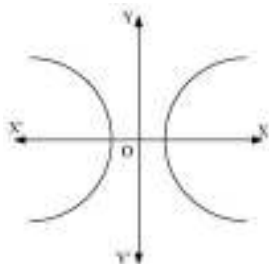
This is the required differential equation.

Question 9:

Form the differential equation of the family of hyperbolas having foci on x -axis and centre at origin.

The equation of the family of hyperbolas with the centre at origin and foci along the x -axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots(1)$$





Differentiating both sides of equation (1) with respect to x , we get:

$$\begin{aligned}\frac{2x}{a^2} - \frac{2yy'}{b^2} &= 0 \\ \Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} &= 0 \quad \dots(2)\end{aligned}$$

Again, differentiating both sides with respect to x , we get:

$$\begin{aligned}\frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} &= 0 \\ \Rightarrow \frac{1}{a^2} &= \frac{1}{b^2} ((y')^2 + yy'')\end{aligned}$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\begin{aligned}\frac{x}{b^2} ((y')^2 + yy'') - \frac{yy'}{b^2} &= 0 \\ \Rightarrow x(y')^2 + xyy'' - yy' &= 0 \\ \Rightarrow xyy'' + x(y')^2 - yy' &= 0\end{aligned}$$

This is the required differential equation.

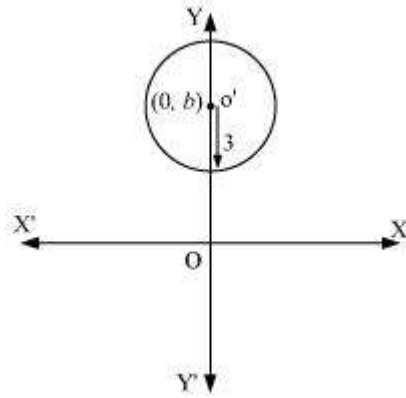
Question 10:

Form the differential equation of the family of circles having centre on y -axis and radius 3 units.

Let the centre of the circle on y -axis be $(0, b)$.

The differential equation of the family of circles with centre at $(0, b)$ and radius 3 is as follows:

$$\begin{aligned}x^2 + (y - b)^2 &= 3^2 \\ \Rightarrow x^2 + (y - b)^2 &= 9 \quad \dots(1)\end{aligned}$$



Differentiating equation (1) with respect to x , we get:

$$\begin{aligned}2x + 2(y - b) \cdot y' &= 0 \\ \Rightarrow (y - b) \cdot y' &= -x \\ \Rightarrow y - b &= \frac{-x}{y'}\end{aligned}$$

Substituting the value of $(y - b)$ in equation (1), we get:

$$\begin{aligned}x^2 + \left(\frac{-x}{y'}\right)^2 &= 9 \\ \Rightarrow x^2 \left[1 + \frac{1}{(y')^2}\right] &= 9 \\ \Rightarrow x^2 \left((y')^2 + 1\right) &= 9(y')^2 \\ \Rightarrow (x^2 - 9)(y')^2 + x^2 &= 0\end{aligned}$$

This is the required differential equation.

Question 11:

Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

A. $\frac{d^2 y}{dx^2} + y = 0$

B. $\frac{d^2 y}{dx^2} - y = 0$



C. $\frac{d^2 y}{dx^2} + 1 = 0$

D. $\frac{d^2 y}{dx^2} - 1 = 0$

The given equation is:

$$y = c_1 e^x + c_2 e^{-x} \quad \dots(1)$$

Differentiating with respect to x , we get:

$$\frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$$

Again, differentiating with respect to x , we get:

$$\begin{aligned} \frac{d^2 y}{dx^2} &= c_1 e^x + c_2 e^{-x} \\ \Rightarrow \frac{d^2 y}{dx^2} &= y \\ \Rightarrow \frac{d^2 y}{dx^2} - y &= 0 \end{aligned}$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Question 12:

Which of the following differential equation has $y = x$ as one of its particular solution?

A. $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$

B. $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = x$

C. $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$



D. $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = 0$

The given equation of curve is $y = x$.

Differentiating with respect to x , we get:

$$\frac{dy}{dx} = 1 \quad \dots(1)$$

Again, differentiating with respect to x , we get:

$$\frac{d^2 y}{dx^2} = 0 \quad \dots(2)$$

Now, on substituting the values of y , $\frac{d^2 y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative **C** is correct.

$$\begin{aligned} \frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy &= 0 - x^2 \cdot 1 + x \cdot x \\ &= -x^2 + x^2 \\ &= 0 \end{aligned}$$

Hence, the correct answer is C.

Exercise – 9.4

Question 1:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

The given differential equation is:



$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \left(\sec^2 \frac{x}{2} - 1 \right)$$

Separating the variables, we get:

$$dy = \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

Now, integrating both sides of this equation, we get:

$$\int dy = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx = \int \sec^2 \frac{x}{2} dx - \int dx$$

$$\Rightarrow y = 2 \tan \frac{x}{2} - x + C$$

This is the required general solution of the given differential equation.

Question 2:

$$\frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$$

The given differential equation is:

$$\frac{dy}{dx} = \sqrt{4 - y^2}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{\sqrt{4 - y^2}} = dx$$

Now, integrating both sides of this equation, we get:

$$\int \frac{dy}{\sqrt{4 - y^2}} = \int dx$$

$$\Rightarrow \sin^{-1} \frac{y}{2} = x + C$$

$$\Rightarrow \frac{y}{2} = \sin(x + C)$$

$$\Rightarrow y = 2 \sin(x + C)$$



This is the required general solution of the given differential equation.

Question 3:

$$\frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} + y &= 1 \\ \Rightarrow dy + y \, dx &= dx \\ \Rightarrow dy &= (1 - y) \, dx\end{aligned}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{1 - y} = dx$$

Now, integrating both sides, we get:

$$\begin{aligned}\int \frac{dy}{1 - y} &= \int dx \\ \Rightarrow \log(1 - y) &= x + \log C \\ \Rightarrow -\log C - \log(1 - y) &= x \\ \Rightarrow \log C(1 - y) &= -x \\ \Rightarrow C(1 - y) &= e^{-x} \\ \Rightarrow 1 - y &= \frac{1}{C} e^{-x} \\ \Rightarrow y &= 1 - \frac{1}{C} e^{-x} \\ \Rightarrow y &= 1 + Ae^{-x} \quad (\text{where } A = -\frac{1}{C})\end{aligned}$$

This is the required general solution of the given differential equation.

Question 4:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

The given differential equation is:



$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

$$\Rightarrow \frac{\sec^2 x \tan y dx + \sec^2 y \tan x dy}{\tan x \tan y} = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

$$\Rightarrow \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides of this equation, we get:

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy \quad \dots(1)$$

Let $\tan x = t$.

$$\therefore \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\text{Now, } \int \frac{\sec^2 x}{\tan x} dx = \int \frac{1}{t} dt.$$

$$= \log t$$

$$= \log(\tan x)$$

$$\text{Similarly, } \int \frac{\sec^2 y}{\tan y} dy = \log(\tan y).$$

Substituting these values in equation (1), we get:

$$\log(\tan x) = -\log(\tan y) + \log C$$

$$\Rightarrow \log(\tan x) = \log\left(\frac{C}{\tan y}\right)$$

$$\Rightarrow \tan x = \frac{C}{\tan y}$$

$$\Rightarrow \tan x \tan y = C$$

This is the required general solution of the given differential equation.

Question 5:



$$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$$

The given differential equation is:

$$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$$

$$\Rightarrow (e^x + e^{-x})dy = (e^x - e^{-x})dx$$

$$\Rightarrow dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx$$

Integrating both sides of this equation, we get:

$$\int dy = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C$$

$$\Rightarrow y = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C \quad \dots(1)$$

Let $(e^x + e^{-x}) = t$.

Differentiating both sides with respect to x , we get:

$$\frac{d}{dx}(e^x + e^{-x}) = \frac{dt}{dx}$$

$$\Rightarrow e^x - e^{-x} = \frac{dt}{dx}$$

$$\Rightarrow (e^x - e^{-x})dx = dt$$

Substituting this value in equation (1), we get:

$$y = \int \frac{1}{t} dt + C$$

$$\Rightarrow y = \log(t) + C$$

$$\Rightarrow y = \log(e^x + e^{-x}) + C$$

This is the required general solution of the given differential equation.

Question 6:



$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

The given differential equation is:

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2) dx$$

Integrating both sides of this equation, we get:

$$\int \frac{dy}{1+y^2} = \int (1+x^2) dx$$

$$\Rightarrow \tan^{-1} y = \int dx + \int x^2 dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

This is the required general solution of the given differential equation.

Question 7:

$$y \log y dx - x dy = 0$$

The given differential equation is:

$$y \log y dx - x dy = 0$$

$$\Rightarrow y \log y dx = x dy$$

$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x} \quad \dots(1)$$

Let $\log y = t$.

$$\therefore \frac{d}{dy}(\log y) = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} dy = dt$$



Substituting this value in equation (1), we get:

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \log t = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

This is the required general solution of the given differential equation.

Question 8:

$$x^5 \frac{dy}{dx} = -y^5$$

The given differential equation is:

$$x^5 \frac{dy}{dx} = -y^5$$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

$$\Rightarrow \frac{dx}{x^5} + \frac{dy}{y^5} = 0$$

Integrating both sides, we get:

$$\int \frac{dx}{x^5} + \int \frac{dy}{y^5} = k \quad (\text{where } k \text{ is any constant})$$

$$\Rightarrow \int x^{-5} dx + \int y^{-5} dy = k$$

$$\Rightarrow \frac{x^{-4}}{-4} + \frac{y^{-4}}{-4} = k$$

$$\Rightarrow x^{-4} + y^{-4} = -4k$$

$$\Rightarrow x^{-4} + y^{-4} = C \quad (C = -4k)$$

This is the required general solution of the given differential equation.

Question 9:



$$\frac{dy}{dx} = \sin^{-1} x$$

The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x$$

$$\Rightarrow dy = \sin^{-1} x \, dx$$

Integrating both sides, we get:

$$\int dy = \int \sin^{-1} x \, dx$$

$$\Rightarrow y = \int (\sin^{-1} x \cdot 1) \, dx$$

$$\Rightarrow y = \sin^{-1} x \cdot \int (1) \, dx - \int \left[\frac{d}{dx} (\sin^{-1} x) \cdot \int (1) \, dx \right] dx$$

$$\Rightarrow y = \sin^{-1} x \cdot x - \int \left(\frac{1}{\sqrt{1-x^2}} \cdot x \right) dx$$

$$\Rightarrow y = x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \quad \dots(1)$$

Let $1-x^2 = t$.

$$\Rightarrow \frac{d}{dx} (1-x^2) = \frac{dt}{dx}$$

$$\Rightarrow -2x = \frac{dt}{dx}$$

$$\Rightarrow x \, dx = -\frac{1}{2} dt$$

Substituting this value in equation (1), we get:

$$y = x \sin^{-1} x + \int \frac{1}{2\sqrt{t}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \int (t)^{-\frac{1}{2}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{t} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$



This is the required general solution of the given differential equation.

Question 10:

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$$

The given differential equation is:

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$$

$$(1 - e^x) \sec^2 y \, dy = -e^x \tan y \, dx$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y} \, dy = \frac{-e^x}{1 - e^x} \, dx$$

Integrating both sides, we get:

$$\int \frac{\sec^2 y}{\tan y} \, dy = \int \frac{-e^x}{1 - e^x} \, dx \quad \dots(1)$$

Let $\tan y = u$.

$$\Rightarrow \frac{d}{dy}(\tan y) = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y = \frac{du}{dy}$$

$$\Rightarrow \sec^2 y \, dy = du$$

$$\therefore \int \frac{\sec^2 y}{\tan y} \, dy = \int \frac{du}{u} = \log u = \log(\tan y)$$

Now, let $1 - e^x = t$.

$$\therefore \frac{d}{dx}(1 - e^x) = \frac{dt}{dx}$$

$$\Rightarrow -e^x = \frac{dt}{dx}$$

$$\Rightarrow -e^x \, dx = dt$$

$$\Rightarrow \int \frac{-e^x}{1 - e^x} \, dx = \int \frac{dt}{t} = \log t = \log(1 - e^x)$$



Substituting the values of $\int \frac{\sec^2 y}{\tan y} dy$ and $\int \frac{-e^x}{1-e^x} dx$ in equation (1), we get:

$$\Rightarrow \log(\tan y) = \log(1 - e^x) + \log C$$

$$\Rightarrow \log(\tan y) = \log[C(1 - e^x)]$$

$$\Rightarrow \tan y = C(1 - e^x)$$

This is the required general solution of the given differential equation.

Question 11:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x; y = 1 \text{ when } x = 0$$

The given differential equation is:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)(x^2+1)} dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \quad \dots(2)$$

$$\Rightarrow \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

Comparing the coefficients of x^2 and x , we get:



$$A + B = 2$$

$$B + C = 1$$

$$A + C = 0$$

Solving these equations, we get:

$$A = \frac{1}{2}, B = \frac{3}{2} \text{ and } C = -\frac{1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2 + x}{(x+1)(x^2 + 1)} = \frac{1}{2} \cdot \frac{1}{(x+1)} + \frac{1}{2} \frac{(3x-1)}{(x^2 + 1)}$$

Therefore, equation (1) becomes:

$$\begin{aligned} \int dy &= \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{3x-1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \cdot \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} \left[2 \log(x+1) + 3 \log(x^2+1) \right] - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} \left[(x+1)^2 (x^2+1)^3 \right] - \frac{1}{2} \tan^{-1} x + C \quad \dots(3) \end{aligned}$$

Now, $y = 1$ when $x = 0$.

$$\Rightarrow 1 = \frac{1}{4} \log(1) - \frac{1}{2} \tan^{-1} 0 + C$$

$$\Rightarrow 1 = \frac{1}{4} \times 0 - \frac{1}{2} \times 0 + C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (3), we get:



$$y = \frac{1}{4} \left[\log(x+1)^2 (x^2+1)^3 \right] - \frac{1}{2} \tan^{-1} x + 1$$

Question 12:

$$x(x^2 - 1) \frac{dy}{dx} = 1; y = 0 \text{ when } x = 2$$

$$x(x^2 - 1) \frac{dy}{dx} = 1$$

$$\Rightarrow dy = \frac{dx}{x(x^2 - 1)}$$

$$\Rightarrow dy = \frac{1}{x(x-1)(x+1)} dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{1}{x(x-1)(x+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \quad \dots(2)$$

$$\begin{aligned} \Rightarrow \frac{1}{x(x-1)(x+1)} &= \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)} \\ &= \frac{(A+B+C)x^2 + (B-C)x - A}{x(x-1)(x+1)} \end{aligned}$$

Comparing the coefficients of x^2 , x , and constant, we get:

$$A = -1$$

$$B - C = 0$$

$$A + B + C = 0$$

Solving these equations, we get $B = \frac{1}{2}$ and $C = \frac{1}{2}$.

Substituting the values of A , B , and C in equation (2), we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$$



Therefore, equation (1) becomes:

$$\begin{aligned} \int dy &= -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x+1} dx \\ \Rightarrow y &= -\log x + \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1) + \log k \\ \Rightarrow y &= \frac{1}{2} \log \left[\frac{k^2(x-1)(x+1)}{x^2} \right] \quad \dots(3) \end{aligned}$$

Now, $y = 0$ when $x = 2$.

$$\begin{aligned} \Rightarrow 0 &= \frac{1}{2} \log \left[\frac{k^2(2-1)(2+1)}{4} \right] \\ \Rightarrow \log \left(\frac{3k^2}{4} \right) &= 0 \\ \Rightarrow \frac{3k^2}{4} &= 1 \\ \Rightarrow 3k^2 &= 4 \\ \Rightarrow k^2 &= \frac{4}{3} \end{aligned}$$

Substituting the value of k^2 in equation (3), we get:

$$\begin{aligned} y &= \frac{1}{2} \log \left[\frac{4(x-1)(x+1)}{3x^2} \right] \\ y &= \frac{1}{2} \log \left[\frac{4(x^2-1)}{3x^2} \right] \end{aligned}$$

Question 13:

$$\cos \left(\frac{dy}{dx} \right) = a \quad (a \in R); y = 1 \text{ when } x = 0$$

$$\cos \left(\frac{dy}{dx} \right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a \, dx$$



Integrating both sides, we get:

$$\begin{aligned}\int dy &= \cos^{-1} a \int dx \\ \Rightarrow y &= \cos^{-1} a \cdot x + C \\ \Rightarrow y &= x \cos^{-1} a + C \quad \dots(1)\end{aligned}$$

Now, $y = 1$ when $x = 0$.

$$\begin{aligned}\Rightarrow 1 &= 0 \cdot \cos^{-1} a + C \\ \Rightarrow C &= 1\end{aligned}$$

Substituting $C = 1$ in equation (1), we get:

$$\begin{aligned}y &= x \cos^{-1} a + 1 \\ \Rightarrow \frac{y-1}{x} &= \cos^{-1} a \\ \Rightarrow \cos\left(\frac{y-1}{x}\right) &= a\end{aligned}$$

Question 14:

$$\frac{dy}{dx} = y \tan x; y = 1 \text{ when } x = 0$$

$$\begin{aligned}\frac{dy}{dx} &= y \tan x \\ \Rightarrow \frac{dy}{y} &= \tan x \, dx\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\int \frac{dy}{y} &= - \int \tan x \, dx \\ \Rightarrow \log y &= \log(\sec x) + \log C \\ \Rightarrow \log y &= \log(C \sec x) \\ \Rightarrow y &= C \sec x \quad \dots(1)\end{aligned}$$

Now, $y = 1$ when $x = 0$.

$$\begin{aligned}\Rightarrow 1 &= C \times \sec 0 \\ \Rightarrow 1 &= C \times 1 \\ \Rightarrow C &= 1\end{aligned}$$



Substituting $C = 1$ in equation (1), we get:

$$y = \sec x$$

Question 15:

Find the equation of a curve passing through the point $(0, 0)$ and whose differential equation is $y' = e^x \sin x$.

The differential equation of the curve is:

$$\begin{aligned}y' &= e^x \sin x \\ \Rightarrow \frac{dy}{dx} &= e^x \sin x \\ \Rightarrow dy &= e^x \sin x\end{aligned}$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x dx \quad \dots(1)$$

$$\text{Let } I = \int e^x \sin x dx.$$

$$\Rightarrow I = \sin x \int e^x dx - \int \left(\frac{d}{dx}(\sin x) \cdot \int e^x dx \right) dx$$

$$\Rightarrow I = \sin x \cdot e^x - \int \cos x \cdot e^x dx$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot \int e^x dx - \int \left(\frac{d}{dx}(\cos x) \cdot \int e^x dx \right) dx \right]$$

$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot e^x - \int (-\sin x) \cdot e^x dx \right]$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow 2I = e^x (\sin x - \cos x)$$

$$\Rightarrow I = \frac{e^x (\sin x - \cos x)}{2}$$

Substituting this value in equation (1), we get:

$$y = \frac{e^x (\sin x - \cos x)}{2} + C \quad \dots(2)$$



Now, the curve passes through point (0, 0).

$$\therefore 0 = \frac{e^0 (\sin 0 - \cos 0)}{2} + C$$

$$\Rightarrow 0 = \frac{1(0-1)}{2} + C$$

$$\Rightarrow C = \frac{1}{2}$$

Substituting $C = \frac{1}{2}$ in equation (2), we get:

$$y = \frac{e^x (\sin x - \cos x)}{2} + \frac{1}{2}$$

$$\Rightarrow 2y = e^x (\sin x - \cos x) + 1$$

$$\Rightarrow 2y - 1 = e^x (\sin x - \cos x)$$

Hence, the required equation of the curve is $2y - 1 = e^x (\sin x - \cos x)$.

Question 16:

For the differential equation $xy \frac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point (1, -1).

The differential equation of the given curve is:

$$xy \frac{dy}{dx} = (x+2)(y+2)$$

$$\Rightarrow \left(\frac{y}{y+2} \right) dy = \left(\frac{x+2}{x} \right) dx$$

$$\Rightarrow \left(1 - \frac{2}{y+2} \right) dy = \left(1 + \frac{2}{x} \right) dx$$

Integrating both sides, we get:



$$\int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$$

$$\Rightarrow \int dy - 2 \int \frac{1}{y+2} dy = \int dx + 2 \int \frac{1}{x} dx$$

$$\Rightarrow y - 2 \log(y+2) = x + 2 \log x + C$$

$$\Rightarrow y - x - C = \log x^2 + \log(y+2)^2$$

$$\Rightarrow y - x - C = \log \left[x^2 (y+2)^2 \right] \quad \dots(1)$$

Now, the curve passes through point (1, -1).

$$\Rightarrow -1 - 1 - C = \log \left[(1)^2 (-1+2)^2 \right]$$

$$\Rightarrow -2 - C = \log 1 = 0$$

$$\Rightarrow C = -2$$

Substituting $C = -2$ in equation (1), we get:

$$y - x + 2 = \log \left[x^2 (y+2)^2 \right]$$

This is the required solution of the given curve.

Question 17:

Find the equation of a curve passing through the point (0, -2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y -coordinate of the point is equal to the x -coordinate of the point.

Let x and y be the x -coordinate and y -coordinate of the curve respectively.

We know that the slope of a tangent to the curve in the coordinate axis is given by the relation,

$$\frac{dy}{dx}$$

According to the given information, we get:



$$y \cdot \frac{dy}{dx} = x$$
$$\Rightarrow y dy = x dx$$

Integrating both sides, we get:

$$\int y dy = \int x dx$$
$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$
$$\Rightarrow y^2 - x^2 = 2C \quad \dots(1)$$

Now, the curve passes through point $(0, -2)$.

$$\therefore (-2)^2 - 0^2 = 2C$$

$$\Rightarrow 2C = 4$$

Substituting $2C = 4$ in equation (1), we get:

$$y^2 - x^2 = 4$$

This is the required equation of the curve.

Question 18:

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2, 1)$.

It is given that (x, y) is the point of contact of the curve and its tangent.

The slope (m_1) of the line segment joining (x, y) and $(-4, -3)$ is $\frac{y+3}{x+4}$.

We know that the slope of the tangent to the curve is given by the relation,

$$\frac{dy}{dx}$$

$$\therefore \text{Slope } (m_2) \text{ of the tangent} = \frac{dy}{dx}$$



According to the given information:

$$\begin{aligned}m_2 &= 2m_1 \\ \Rightarrow \frac{dy}{dx} &= \frac{2(y+3)}{x+4} \\ \Rightarrow \frac{dy}{y+3} &= \frac{2dx}{x+4}\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\int \frac{dy}{y+3} &= 2 \int \frac{dx}{x+4} \\ \Rightarrow \log(y+3) &= 2 \log(x+4) + \log C \\ \Rightarrow \log(y+3) &= \log C(x+4)^2 \\ \Rightarrow y+3 &= C(x+4)^2 \quad \dots(1)\end{aligned}$$

This is the general equation of the curve.

It is given that it passes through point $(-2, 1)$.

$$\begin{aligned}\Rightarrow 1+3 &= C(-2+4)^2 \\ \Rightarrow 4 &= 4C \\ \Rightarrow C &= 1\end{aligned}$$

Substituting $C = 1$ in equation (1), we get:

$$y + 3 = (x + 4)^2$$

This is the required equation of the curve.

Question 19:

The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Let the rate of change of the volume of the balloon be k (where k is a constant).



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$$\Rightarrow \frac{dv}{dt} = k$$

$$\Rightarrow \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \right) = k$$

$$\Rightarrow \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k dt$$

$$\left[\text{Volume of sphere} = \frac{4}{3} \pi r^3 \right]$$

Integrating both sides, we get:

$$4\pi \int r^2 dr = k \int dt$$

$$\Rightarrow 4\pi \cdot \frac{r^3}{3} = kt + C$$

$$\Rightarrow 4\pi r^3 = 3(kt + C) \quad \dots(1)$$

Now, at $t = 0, r = 3$:

$$\Rightarrow 4\pi \times 3^3 = 3(k \times 0 + C)$$

$$\Rightarrow 108\pi = 3C$$

$$\Rightarrow C = 36\pi$$

At $t = 3, r = 6$:

$$\Rightarrow 4\pi \times 6^3 = 3(k \times 3 + C)$$

$$\Rightarrow 864\pi = 3(3k + 36\pi)$$

$$\Rightarrow 3k = -288\pi - 36\pi = 252\pi$$

$$\Rightarrow k = 84\pi$$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$

$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t + 27)^{\frac{1}{3}}$.



Question 20:

In a bank, principal increases continuously at the rate of $r\%$ per year. Find the value of r if Rs 100 doubles itself in 10 years ($\log_e 2 = 0.6931$).

Let p , t , and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of $r\%$ per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100} \int dt$$

$$\Rightarrow \log p = \frac{rt}{100} + k$$

$$\Rightarrow p = e^{\frac{rt}{100} + k} \quad \dots(1)$$

It is given that when $t = 0$, $p = 100$.

$$\Rightarrow 100 = e^k \dots (2)$$

Now, if $t = 10$, then $p = 2 \times 100 = 200$.

Therefore, equation (1) becomes:

$$200 = e^{\frac{r}{10} + k}$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot e^k$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot 100 \quad \text{(From (2))}$$

$$\Rightarrow e^{\frac{r}{10}} = 2$$

$$\Rightarrow \frac{r}{10} = \log_e 2$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$



Hence, the value of r is 6.93%.

Question 21:

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100} \right) p$$

$$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20} \int dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20} + C} \quad \dots(1)$$

Now, when $t = 0$, $p = 1000$.

$$\Rightarrow 1000 = e^C \dots (2)$$

At $t = 10$, equation (1) becomes:

$$p = e^{\frac{1}{2} + C}$$

$$\Rightarrow p = e^{0.5} \times e^C$$

$$\Rightarrow p = 1.648 \times 1000$$

$$\Rightarrow p = 1648$$



Hence, after 10 years the amount will worth Rs 1648.

Question 22:

In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Let y be the number of bacteria at any instant t .

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\begin{aligned}\therefore \frac{dy}{dt} &\propto y \\ \Rightarrow \frac{dy}{dt} &= ky \text{ (where } k \text{ is a constant)} \\ \Rightarrow \frac{dy}{y} &= k dt\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\int \frac{dy}{y} &= k \int dt \\ \Rightarrow \log y &= kt + C \quad \dots(1)\end{aligned}$$

Let y_0 be the number of bacteria at $t = 0$.

$$\Rightarrow \log y_0 = C$$

Substituting the value of C in equation (1), we get:

$$\begin{aligned}\log y &= kt + \log y_0 \\ \Rightarrow \log y - \log y_0 &= kt \\ \Rightarrow \log \left(\frac{y}{y_0} \right) &= kt \\ \Rightarrow kt &= \log \left(\frac{y}{y_0} \right) \quad \dots(2)\end{aligned}$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.



$$\Rightarrow y = \frac{110}{100} y_0$$
$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \quad \dots(3)$$

Substituting this value in equation (2), we get:

$$k \cdot 2 = \log\left(\frac{11}{10}\right)$$
$$\Rightarrow k = \frac{1}{2} \log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2} \log\left(\frac{11}{10}\right) \cdot t = \log\left(\frac{y}{y_0}\right)$$
$$\Rightarrow t = \frac{2 \log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \quad \dots(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1 .

$$\Rightarrow y = 2y_0 \text{ at } t = t_1$$

From equation (4), we get:

$$t_1 = \frac{2 \log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Question 23:

The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is



A. $e^x + e^{-y} = C$

B. $e^x + e^y = C$

C. $e^{-x} + e^y = C$

D. $e^{-x} + e^{-y} = C$

$$\frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + k$$

$$\Rightarrow e^x + e^{-y} = -k$$

$$\Rightarrow e^x + e^{-y} = c \quad (c = -k)$$

Hence, the correct answer is A.

Exercise – 9.5

Question 1:

$$(x^2 + xy) dy = (x^2 + y^2) dx$$

The given differential equation i.e., $(x^2 + xy) dy = (x^2 + y^2) dx$ can be written as:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x^2 + y^2}{x^2 + xy}.$$

$$\text{Now, } F(\lambda x, \lambda y) = \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)} = \frac{x^2 + y^2}{x^2 + xy} = \lambda^0 \cdot F(x, y)$$



This shows that equation (1) is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

Differentiating both sides with respect to x , we get:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of v and $\frac{dy}{dx}$ in equation (1), we get:

$$\begin{aligned}v + x \frac{dv}{dx} &= \frac{x^2 + (vx)^2}{x^2 + x(vx)} \\ \Rightarrow v + x \frac{dv}{dx} &= \frac{1 + v^2}{1 + v} \\ \Rightarrow x \frac{dv}{dx} &= \frac{1 + v^2}{1 + v} - v = \frac{(1 + v^2) - v(1 + v)}{1 + v} \\ \Rightarrow x \frac{dv}{dx} &= \frac{1 - v}{1 + v} \\ \Rightarrow \left(\frac{1 + v}{1 - v} \right) &= dv = \frac{dx}{x} \\ \Rightarrow \left(\frac{2 - 1 + v}{1 - v} \right) dv &= \frac{dx}{x} \\ \Rightarrow \left(\frac{2}{1 - v} - 1 \right) dv &= \frac{dx}{x}\end{aligned}$$

Integrating both sides, we get:



$$-2 \log(1-v) - v = \log x - \log k$$

$$\Rightarrow v = -2 \log(1-v) - \log x + \log k$$

$$\Rightarrow v = \log \left[\frac{k}{x(1-v)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[\frac{k}{x \left(1 - \frac{y}{x}\right)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[\frac{kx}{(x-y)^2} \right]$$

$$\Rightarrow \frac{kx}{(x-y)^2} = e^{\frac{y}{x}}$$

$$\Rightarrow (x-y)^2 = kxe^{-\frac{y}{x}}$$

This is the required solution of the given differential equation.

Question 2:

$$y' = \frac{x+y}{x}$$

The given differential equation is:

$$y' = \frac{x+y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x+y}{x}.$$

$$\text{Now, } F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x} = \frac{x+y}{x} = \lambda^0 F(x, y)$$

Thus, the given equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$



Differentiating both sides with respect to x , we get:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x + vx}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 + v$$

$$x \frac{dv}{dx} = 1$$

$$\Rightarrow dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$v = \log x + C$$

$$\Rightarrow \frac{y}{x} = \log x + C$$

$$\Rightarrow y = x \log x + Cx$$

This is the required solution of the given differential equation.

Question 3:

$$(x - y)dy - (x + y)dx = 0$$

The given differential equation is:

$$(x - y)dy - (x + y)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x + y}{x - y}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \frac{x + y}{x - y} = \lambda^0 \cdot F(x, y)$$



Thus, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{1 + v}{1 - v}$$

$$x \frac{dv}{dx} = \frac{1 + v}{1 - v} - v = \frac{1 + v - v(1 - v)}{1 - v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{1 - v}$$

$$\Rightarrow \frac{1 - v}{(1 + v^2)} dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{1 + v^2} - \frac{v}{1 - v^2} \right) dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\tan^{-1} v - \frac{1}{2} \log(1 + v^2) = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log \left[1 + \left(\frac{y}{x} \right)^2 \right] = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log \left(\frac{x^2 + y^2}{x^2} \right) = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} [\log(x^2 + y^2) - \log x^2] = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log(x^2 + y^2) + C$$



This is the required solution of the given differential equation.

Question 4:

$$(x^2 - y^2)dx + 2xy dy = 0$$

The given differential equation is:

$$(x^2 - y^2)dx + 2xy dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x^2 - y^2)}{2xy} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{-(x^2 - y^2)}{2xy}.$$

$$\therefore F(\lambda x, \lambda y) = \left[\frac{(\lambda x)^2 - (\lambda y)^2}{2(\lambda x)(\lambda y)} \right] = \frac{-(x^2 - y^2)}{2xy} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:



$$v + x \frac{dv}{dx} = - \left[\frac{x^2 - (vx)^2}{2x \cdot (vx)} \right]$$

$$v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = - \frac{(1 + v^2)}{2v}$$

$$\Rightarrow \frac{2v}{1 + v^2} dv = - \frac{dx}{x}$$

Integrating both sides, we get:

$$\log(1 + v^2) = -\log x + \log C = \log \frac{C}{x}$$

$$\Rightarrow 1 + v^2 = \frac{C}{x}$$

$$\Rightarrow \left[1 + \frac{y^2}{x^2} \right] = \frac{C}{x}$$

$$\Rightarrow x^2 + y^2 = Cx$$

This is the required solution of the given differential equation.

Question 5:

$$x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$$

The given differential equation is:

$$x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$$



$$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{x^2 - 2y^2 + xy}{x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{(\lambda x)^2 - 2(\lambda y)^2 + (\lambda x)(\lambda y)}{(\lambda x)^2} = \frac{x^2 - 2y^2 + xy}{x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x^2 - 2(vx)^2 + x \cdot (vx)}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 - 2v^2 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1 - 2v^2$$

$$\Rightarrow \frac{dv}{1 - 2v^2} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{dv}{\frac{1}{2} - v^2} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \cdot \left[\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} \right] = \frac{dx}{x}$$

Integrating both sides, we get:



$$\frac{1}{2} \cdot \frac{1}{2 \times \frac{1}{\sqrt{2}}} \log \left| \frac{\frac{1}{\sqrt{2}} + v}{\frac{1}{\sqrt{2}} - v} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\frac{1}{\sqrt{2}} + \frac{y}{x}}{\frac{1}{\sqrt{2}} - \frac{y}{x}} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \sqrt{2}y}{x - \sqrt{2}y} \right| = \log|x| + C$$

This is the required solution for the given differential equation.

Question 6:

$$x dy - y dx = \sqrt{x^2 + y^2} dx$$

$$x dy - y dx = \sqrt{x^2 + y^2} dx$$

$$\Rightarrow x dy = \left[y + \sqrt{x^2 + y^2} \right] dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{y + \sqrt{x^2 + y^2}}{x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y + \sqrt{(\lambda x)^2 + (\lambda y)^2}}{\lambda x^2} = \frac{y + \sqrt{x^2 + y^2}}{x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$



Substituting the values of v and $\frac{dy}{dx}$ in equation (1), we get:

$$\begin{aligned}v + x \frac{dv}{dx} &= \frac{vx + \sqrt{x^2 + (vx)^2}}{x} \\ \Rightarrow v + x \frac{dv}{dx} &= v + \sqrt{1 + v^2} \\ \Rightarrow \frac{dv}{\sqrt{1 + v^2}} &= \frac{dx}{x}\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\log \left| v + \sqrt{1 + v^2} \right| &= \log |x| + \log C \\ \Rightarrow \log \left| \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} \right| &= \log |Cx| \\ \Rightarrow \log \left| \frac{y + \sqrt{x^2 + y^2}}{x} \right| &= \log |Cx| \\ \Rightarrow y + \sqrt{x^2 + y^2} &= Cx^2\end{aligned}$$

This is the required solution of the given differential equation.

Question 7:

$$\left\{ x \cos \left(\frac{y}{x} \right) + y \sin \left(\frac{y}{x} \right) \right\} y dx = \left\{ y \sin \left(\frac{y}{x} \right) - x \cos \left(\frac{y}{x} \right) \right\} x dy$$

The given differential equation is:



$$\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$$

$$\frac{dy}{dx} = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x}.$$

$$\begin{aligned} \therefore F(\lambda x, \lambda y) &= \frac{\left\{ \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda y}{\left\{ \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda x} \\ &= \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x} \\ &= \lambda^0 \cdot F(x, y) \end{aligned}$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x = \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:



$$\begin{aligned}
 v+x \frac{dv}{dx} &= \frac{(x \cos v + vx \sin v) \cdot vx}{(vx \sin v - x \cos v) \cdot x} \\
 \Rightarrow v+x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} \\
 \Rightarrow x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} - v \\
 \Rightarrow x \frac{dv}{dx} &= \frac{v \cos v + v^2 \sin v - v^2 \sin v + v \cos v}{v \sin v - \cos v} \\
 \Rightarrow x \frac{dv}{dx} &= \frac{2v \cos v}{v \sin v - \cos v} \\
 \Rightarrow \left[\frac{v \sin v - \cos v}{v \cos v} \right] dv &= \frac{2dx}{x} \\
 \Rightarrow \left(\tan v - \frac{1}{v} \right) dv &= \frac{2dx}{x}
 \end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}
 \log(\sec v) - \log v &= 2 \log x + \log C \\
 \Rightarrow \log\left(\frac{\sec v}{v}\right) &= \log(Cx^2) \\
 \Rightarrow \left(\frac{\sec v}{v}\right) &= Cx^2 \\
 \Rightarrow \sec v &= Cx^2 v \\
 \Rightarrow \sec\left(\frac{y}{x}\right) &= C \cdot x^2 \cdot \frac{y}{x} \\
 \Rightarrow \sec\left(\frac{y}{x}\right) &= Cxy \\
 \Rightarrow \cos\left(\frac{y}{x}\right) &= \frac{1}{Cxy} = \frac{1}{C} \cdot \frac{1}{xy} \\
 \Rightarrow xy \cos\left(\frac{y}{x}\right) &= k \quad \left(k = \frac{1}{C}\right)
 \end{aligned}$$

This is the required solution of the given differential equation.

Question 8:



$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow x \frac{dy}{dx} = y - x \sin\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y - \lambda x \sin\left(\frac{\lambda y}{\lambda x}\right)}{\lambda x} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{vx - x \sin v}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \sin v$$

$$\Rightarrow -\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\Rightarrow \operatorname{cosec} v \, dv = -\frac{dx}{x}$$



Integrating both sides, we get:

$$\log|\operatorname{cosec} v - \cot v| = -\log x + \log C = \log \frac{C}{x}$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{C}{x}$$

$$\Rightarrow \frac{1}{\sin\left(\frac{y}{x}\right)} - \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{C}{x}$$

$$\Rightarrow x \left[1 - \cos\left(\frac{y}{x}\right) \right] = C \sin\left(\frac{y}{x}\right)$$

This is the required solution of the given differential equation.

Question 9:

$$ydx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$$

$$ydx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$$

$$\Rightarrow ydx = \left[2x - x \log\left(\frac{y}{x}\right) \right] dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x - x \log\left(\frac{y}{x}\right)} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{y}{2x - x \log\left(\frac{y}{x}\right)}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y}{2(\lambda x) - (\lambda x) \log\left(\frac{\lambda y}{\lambda x}\right)} = \frac{y}{2x - \log\left(\frac{y}{x}\right)} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:



$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{vx}{2x - x \log v}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - 2v + v \log v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \log v - v}{2 - \log v}$$

$$\Rightarrow \frac{2 - \log v}{v(\log v - 1)} dv = \frac{dx}{x}$$

$$\Rightarrow \left[\frac{1 + (1 - \log v)}{v(\log v - 1)} \right] dv = \frac{dx}{x}$$

$$\Rightarrow \left[\frac{1}{v(\log v - 1)} - \frac{1}{v} \right] dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{1}{v(\log v - 1)} dv - \int \frac{1}{v} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{dv}{v(\log v - 1)} - \log v = \log x + \log C \quad \dots(2)$$

$$\Rightarrow \text{Let } \log v - 1 = t$$

$$\Rightarrow \frac{d}{dv}(\log v - 1) = \frac{dt}{dv}$$

$$\Rightarrow \frac{1}{v} = \frac{dt}{dv}$$

$$\Rightarrow \frac{dv}{v} = dt$$

Therefore, equation (1) becomes:



$$\Rightarrow \int \frac{dt}{t} - \log v = \log x + \log C$$

$$\Rightarrow \log t - \log\left(\frac{y}{x}\right) = \log(Cx)$$

$$\Rightarrow \log\left[\log\left(\frac{y}{x}\right) - 1\right] - \log\left(\frac{y}{x}\right) = \log(Cx)$$

$$\Rightarrow \log\left[\frac{\log\left(\frac{y}{x}\right) - 1}{\frac{y}{x}}\right] = \log(Cx)$$

$$\Rightarrow \frac{x}{y}\left[\log\left(\frac{y}{x}\right) - 1\right] = Cx$$

$$\Rightarrow \log\left(\frac{y}{x}\right) - 1 = Cy$$

This is the required solution of the given differential equation.

Question 10:

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

$$\Rightarrow \left(1 + e^{\frac{x}{y}}\right) dx = -e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy$$



$$\Rightarrow \frac{dx}{dy} = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$$

$$\therefore F(\lambda x, \lambda y) = \frac{-e^{\frac{\lambda x}{\lambda y}} \left(1 - \frac{\lambda x}{\lambda y}\right)}{1 + e^{\frac{\lambda x}{\lambda y}}} = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$x = vy$$

$$\Rightarrow \frac{d}{dy}(x) = \frac{d}{dy}(vy)$$

$$\Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$$

Substituting the values of x and $\frac{dx}{dy}$ in equation (1), we get:

$$v + y \frac{dv}{dy} = \frac{-e^v (1 - v)}{1 + e^v}$$

$$\Rightarrow y \frac{dv}{dy} = \frac{-e^v + ve^v}{1 + e^v} - v$$

$$\Rightarrow y \frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1 + e^v}$$

$$\Rightarrow y \frac{dv}{dy} = - \left[\frac{v + e^v}{1 + e^v} \right]$$

$$\Rightarrow \left[\frac{1 + e^v}{v + e^v} \right] dv = - \frac{dy}{y}$$

Integrating both sides, we get:



$$\Rightarrow \log(v + e^v) = -\log y + \log C = \log\left(\frac{C}{y}\right)$$

$$\Rightarrow \left[\frac{x}{y} + e^{\frac{x}{y}}\right] = \frac{C}{y}$$

$$\Rightarrow x + ye^{\frac{x}{y}} = C$$

This is the required solution of the given differential equation.

Question 11:

$$(x + y)dy + (x - y)dx = 0; y = 1 \text{ when } x = 1$$

$$(x + y)dy + (x - y)dx = 0$$

$$\Rightarrow (x + y)dy = -(x - y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x - y)}{x + y} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{-(x - y)}{x + y}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{-(\lambda x - \lambda y)}{\lambda x + \lambda y} = \frac{-(x - y)}{x + y} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:



$$\begin{aligned}v+x \frac{dv}{dx} &= \frac{-(x-vx)}{x+vx} \\ \Rightarrow v+x \frac{dv}{dx} &= \frac{v-1}{v+1} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v-1}{v+1} - v = \frac{v-1-v(v+1)}{v+1} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v-1-v^2-v}{v+1} = \frac{-(1+v^2)}{v+1} \\ \Rightarrow \frac{(v+1)}{1+v^2} dv &= -\frac{dx}{x} \\ \Rightarrow \left[\frac{v}{1+v^2} + \frac{1}{1+v^2} \right] dv &= -\frac{dx}{x}\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\frac{1}{2} \log(1+v^2) + \tan^{-1} v &= -\log x + k \\ \Rightarrow \log(1+v^2) + 2 \tan^{-1} v &= -2 \log x + 2k \\ \Rightarrow \log[(1+v^2) \cdot x^2] + 2 \tan^{-1} v &= 2k \\ \Rightarrow \log\left[\left(1 + \frac{y^2}{x^2}\right) \cdot x^2\right] + 2 \tan^{-1} \frac{y}{x} &= 2k \\ \Rightarrow \log(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} &= 2k \quad \dots(2)\end{aligned}$$

Now, $y = 1$ at $x = 1$.

$$\begin{aligned}\Rightarrow \log 2 + 2 \tan^{-1} 1 &= 2k \\ \Rightarrow \log 2 + 2 \times \frac{\pi}{4} &= 2k \\ \Rightarrow \frac{\pi}{2} + \log 2 &= 2k\end{aligned}$$

Substituting the value of $2k$ in equation (2), we get:

$$\log(x^2 + y^2) + 2 \tan^{-1} \left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$$

This is the required solution of the given differential equation.



Question 12:

$$x^2 dy + (xy + y^2) dx = 0; y = 1 \text{ when } x = 1$$

$$x^2 dy + (xy + y^2) dx = 0$$

$$\Rightarrow x^2 dy = -(xy + y^2) dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(xy + y^2)}{x^2} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{-(xy + y^2)}{x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{[\lambda x \cdot \lambda y + (\lambda y)^2]}{(\lambda x)^2} = \frac{-(xy + y^2)}{x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:



$$v + x \frac{dv}{dx} = \frac{-[x \cdot vx + (vx)^2]}{x^2} = -v - v^2$$

$$\Rightarrow x \frac{dv}{dx} = -v^2 - 2v = -v(v+2)$$

$$\Rightarrow \frac{dv}{v(v+2)} = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \left[\frac{(v+2) - v}{v(v+2)} \right] dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \left[\frac{1}{v} - \frac{1}{v+2} \right] dv = -\frac{dx}{x}$$

Integrating both sides, we get:

$$\frac{1}{2} [\log v - \log(v+2)] = -\log x + \log C$$

$$\Rightarrow \frac{1}{2} \log \left(\frac{v}{v+2} \right) = \log \frac{C}{x}$$

$$\Rightarrow \frac{v}{v+2} = \left(\frac{C}{x} \right)^2$$

$$\Rightarrow \frac{\frac{y}{x}}{\frac{y}{x} + 2} = \left(\frac{C}{x} \right)^2$$

$$\Rightarrow \frac{y}{y+2x} = \frac{C^2}{x^2}$$

$$\Rightarrow \frac{x^2 y}{y+2x} = C^2 \quad \dots(2)$$

Now, $y = 1$ at $x = 1$.

$$\Rightarrow \frac{1}{1+2} = C^2$$

$$\Rightarrow C^2 = \frac{1}{3}$$

Substituting $C^2 = \frac{1}{3}$ in equation (2), we get:



$$\frac{x^2 y}{y+2x} = \frac{1}{3}$$

$$\Rightarrow y+2x = 3x^2 y$$

This is the required solution of the given differential equation.

Question 13:

$$\left[x \sin^2 \left(\frac{x}{y} - y \right) \right] dx + x dy = 0; y = \frac{\pi}{4} \text{ when } x = 1$$

$$\left[x \sin^2 \left(\frac{y}{x} \right) - y \right] dx + x dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{- \left[x \sin^2 \left(\frac{y}{x} \right) - y \right]}{x} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{- \left[x \sin^2 \left(\frac{y}{x} \right) - y \right]}{x}$$

$$\therefore F(\lambda x, \lambda y) = \frac{- \left[\lambda x \cdot \sin^2 \left(\frac{\lambda y}{\lambda x} \right) - \lambda y \right]}{\lambda x} = \frac{- \left[x \sin^2 \left(\frac{y}{x} \right) - y \right]}{x} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve this differential equation, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x = \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:



$$\begin{aligned}v + x \frac{dv}{dx} &= \frac{-[x \sin^2 v - vx]}{x} \\ \Rightarrow v + x \frac{dv}{dx} &= -[\sin^2 v - v] = v - \sin^2 v \\ \Rightarrow x \frac{dv}{dx} &= -\sin^2 v \\ \Rightarrow \frac{dv}{\sin^2 v} &= -\frac{dx}{x} \\ \Rightarrow \operatorname{cosec}^2 v dv &= -\frac{dx}{x}\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}-\cot v &= -\log|x| - C \\ \Rightarrow \cot v &= \log|x| + C \\ \Rightarrow \cot\left(\frac{y}{x}\right) &= \log|x| + \log C \\ \Rightarrow \cot\left(\frac{y}{x}\right) &= \log|Cx| \quad \dots(2)\end{aligned}$$

Now, $y = \frac{\pi}{4}$ at $x = 1$.

$$\begin{aligned}\Rightarrow \cot\left(\frac{\pi}{4}\right) &= \log|C| \\ \Rightarrow 1 &= \log C \\ \Rightarrow C &= e^1 = e\end{aligned}$$

Substituting $C = e$ in equation (2), we get:

$$\cot\left(\frac{y}{x}\right) = \log|ex|$$

This is the required solution of the given differential equation.

Question 14:

$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0; y = 0 \text{ when } x = 1$$



$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right) \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right).$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} - \operatorname{cosec}\left(\frac{\lambda y}{\lambda x}\right)$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right) = F(x, y) = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = v - \operatorname{cosec} v$$

$$\Rightarrow -\frac{dv}{\operatorname{cosec} v} = -\frac{dx}{x}$$

$$\Rightarrow -\sin v dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\cos v = \log x + \log C = \log |Cx|$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log |Cx| \quad \dots(2)$$

This is the required solution of the given differential equation.



Now, $y = 0$ at $x = 1$.

$$\Rightarrow \cos(0) = \log C$$

$$\Rightarrow 1 = \log C$$

$$\Rightarrow C = e^1 = e$$

Substituting $C = e$ in equation (2), we get:

$$\cos\left(\frac{y}{x}\right) = \log|ex|$$

This is the required solution of the given differential equation.

Question 15:

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2 \text{ when } x = 1$$

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$$

$$\Rightarrow 2x^2 \frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad \dots(1)$$

$$\text{Let } F(x, y) = \frac{2xy + y^2}{2x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{2(\lambda x)(\lambda y) + (\lambda y)^2}{2(\lambda x)^2} = \frac{2xy + y^2}{2x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$



Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get:

$$\begin{aligned}v + x \frac{dv}{dx} &= \frac{2x(vx) + (vx)^2}{2x^2} \\ \Rightarrow v + x \frac{dv}{dx} &= \frac{2v + v^2}{2} \\ \Rightarrow v + x \frac{dv}{dx} &= v + \frac{v^2}{2} \\ \Rightarrow \frac{2}{v^2} dv &= \frac{dx}{x}\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}2 \cdot \frac{v^{-2+1}}{-2+1} &= \log|x| + C \\ \Rightarrow -\frac{2}{v} &= \log|x| + C \\ \Rightarrow -\frac{2}{\frac{y}{x}} &= \log|x| + C \\ \Rightarrow -\frac{2x}{y} &= \log|x| + C \quad \dots(2)\end{aligned}$$

Now, $y = 2$ at $x = 1$.

$$\begin{aligned}\Rightarrow -1 &= \log(1) + C \\ \Rightarrow C &= -1\end{aligned}$$

Substituting $C = -1$ in equation (2), we get:

$$\begin{aligned}-\frac{2x}{y} &= \log|x| - 1 \\ \Rightarrow \frac{2x}{y} &= 1 - \log|x| \\ \Rightarrow y &= \frac{2x}{1 - \log|x|}, (x \neq 0, x \neq e)\end{aligned}$$

This is the required solution of the given differential equation.



Question 16:

A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

A. $y = vx$

B. $v = yx$

C. $x = vy$

D. $x = v$

For solving the homogeneous equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$, we need to make the substitution as $x = vy$.

Hence, the correct answer is C.

Question 17:

Which of the following is a homogeneous differential equation?

A. $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

B. $(xy)dx - (x^3 + y^3)dy = 0$

C. $(x^3 + 2y^2)dx + 2xy dy = 0$

D. $y^2 dx + (x^2 - xy^2 - y^2)dy = 0$

Function $F(x, y)$ is said to be the homogenous function of degree n , if

$$F(\lambda x, \lambda y) = \lambda^n F(x, y) \text{ for any non-zero constant } (\lambda).$$

Consider the equation given in alternative D:



$$y^2 dx + (x^2 - xy - y^2) dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2 - xy - y^2} = \frac{y^2}{y^2 + xy - x^2}$$

$$\text{Let } F(x, y) = \frac{y^2}{y^2 + xy - x^2}.$$

$$\begin{aligned}\Rightarrow F(\lambda x, \lambda y) &= \frac{(\lambda y)^2}{(\lambda y)^2 + (\lambda x)(\lambda y) - (\lambda x)^2} \\ &= \frac{\lambda^2 y^2}{\lambda^2 (y^2 + xy - x^2)} \\ &= \lambda^0 \left(\frac{y^2}{y^2 + xy - x^2} \right) \\ &= \lambda^0 \cdot F(x, y)\end{aligned}$$

Hence, the differential equation given in alternative **D** is a homogenous equation.

Exercise – 9.6

Question 1:

$$\frac{dy}{dx} + 2y = \sin x$$

The given differential equation is $\frac{dy}{dx} + 2y = \sin x$.

This is in the form of $\frac{dy}{dx} + py = Q$ (where $p = 2$ and $Q = \sin x$).

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int 2 dx} = e^{2x}.$$

The solution of the given differential equation is given by the relation,



$$y(\text{I.F.}) = \int(Q \times \text{I.F.}) dx + C$$

$$\Rightarrow ye^{2x} = \int \sin x \cdot e^{2x} dx + C \quad \dots(1)$$

$$\text{Let } I = \int \sin x \cdot e^{2x}.$$

$$\Rightarrow I = \sin x \cdot \int e^{2x} dx - \int \left(\frac{d}{dx}(\sin x) \cdot \int e^{2x} dx \right) dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \left(\cos x \cdot \frac{e^{2x}}{2} \right) dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \int e^{2x} - \int \left(\frac{d}{dx}(\cos x) \cdot \int e^{2x} dx \right) dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int \left((-\sin x) \cdot \frac{e^{2x}}{2} \right) dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} \int (\sin x \cdot e^{2x}) dx$$

$$\Rightarrow I = \frac{e^{2x}}{4} (2 \sin x - \cos x) - \frac{1}{4} I$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x}}{4} (2 \sin x - \cos x)$$

$$\Rightarrow I = \frac{e^{2x}}{5} (2 \sin x - \cos x)$$

Therefore, equation (1) becomes:

$$ye^{2x} = \frac{e^{2x}}{5} (2 \sin x - \cos x) + C$$

$$\Rightarrow y = \frac{1}{5} (2 \sin x - \cos x) + Ce^{-2x}$$

This is the required general solution of the given differential equation.

Question 2:

$$\frac{dy}{dx} + 3y = e^{-2x}$$

$$\frac{dy}{dx} + py = Q \quad (\text{where } p = 3 \text{ and } Q = e^{-2x}).$$

The given differential equation is



$$\text{Now, I.F} = e^{\int p dx} = e^{\int 3 dx} = e^{3x}.$$

The solution of the given differential equation is given by the relation,

$$\begin{aligned} y(\text{I.F.}) &= \int (Q \times \text{I.F.}) dx + C \\ \Rightarrow ye^{3x} &= \int (e^{-2x} \times e^{3x}) + C \\ \Rightarrow ye^{3x} &= \int e^x dx + C \\ \Rightarrow ye^{3x} &= e^x + C \\ \Rightarrow y &= e^{-2x} + Ce^{-3x} \end{aligned}$$

This is the required general solution of the given differential equation.

Question 3:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

The given differential equation is:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{1}{x} \text{ and } Q = x^2)$$

$$\text{Now, I.F} = e^{\int p dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x.$$

The solution of the given differential equation is given by the relation,

$$\begin{aligned} y(\text{I.F.}) &= \int (Q \times \text{I.F.}) dx + C \\ \Rightarrow y(x) &= \int (x^2 \cdot x) dx + C \\ \Rightarrow xy &= \int x^3 dx + C \\ \Rightarrow xy &= \frac{x^4}{4} + C \end{aligned}$$

This is the required general solution of the given differential equation.

Question 4:

$$\frac{dy}{dx} + \sec xy = \tan x \left(0 \leq x < \frac{\pi}{2} \right)$$



The given differential equation is:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \sec x \text{ and } Q = \tan x \text{)}$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x.$$

The general solution of the given differential equation is given by the relation,

$$\begin{aligned} y(\text{I.F.}) &= \int(Q \times \text{I.F.}) dx + C \\ \Rightarrow y(\sec x + \tan x) &= \int \tan x (\sec x + \tan x) dx + C \\ \Rightarrow y(\sec x + \tan x) &= \int \sec x \tan x dx + \int \tan^2 x dx + C \\ \Rightarrow y(\sec x + \tan x) &= \sec x + \int (\sec^2 x - 1) dx + C \\ \Rightarrow y(\sec x + \tan x) &= \sec x + \tan x - x + C \end{aligned}$$

Question 5:

$$\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \leq x < \frac{\pi}{2} \right)$$

The given differential equation is:

$$\begin{aligned} \cos^2 x \frac{dy}{dx} + y &= \tan x \\ \Rightarrow \frac{dy}{dx} + \sec^2 x \cdot y &= \sec^2 x \tan x \end{aligned}$$

This equation is in the form of:

$$\begin{aligned} \frac{dy}{dx} + py &= Q \text{ (where } p = \sec^2 x \text{ and } Q = \sec^2 x \tan x \text{)} \\ \text{Now, I.F.} &= e^{\int p dx} = e^{\int \sec^2 x dx} = e^{\tan x}. \end{aligned}$$

The general solution of the given differential equation is given by the relation,



$$y(\text{I.F.}) = \int(Q \times \text{I.F.}) dx + C$$

$$\Rightarrow y \cdot e^{\tan x} = \int e^{\tan x} \cdot \sec^2 x \tan x dx + C \quad \dots(1)$$

Let $\tan x = t$.

$$\Rightarrow \frac{d}{dx}(\tan x) = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec^2 x dx = dt$$

Therefore, equation (1) becomes:

$$y \cdot e^{\tan x} = \int(e^t \cdot t) dt + C$$

$$\Rightarrow y \cdot e^{\tan x} = \int(t \cdot e^t) dt + C$$

$$\Rightarrow y \cdot e^{\tan x} = t \cdot \int e^t dt - \int\left(\frac{d}{dt}(t) \cdot \int e^t dt\right) dt + C$$

$$\Rightarrow y \cdot e^{\tan x} = t \cdot e^t - \int e^t dt + C$$

$$\Rightarrow ye^{\tan x} = (t-1)e^t + C$$

$$\Rightarrow ye^{\tan x} = (\tan x - 1)e^{\tan x} + C$$

$$\Rightarrow y = (\tan x - 1) + Ce^{-\tan x}$$

Question 6:

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

The given differential equation is:

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x \log x$$

This equation is in the form of a linear differential equation as:

$$\frac{dy}{dx} + py = Q \quad (\text{where } p = \frac{2}{x} \text{ and } Q = x \log x)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2.$$



The general solution of the given differential equation is given by the relation,

$$\begin{aligned}
 y(\text{I.F.}) &= \int(Q \times \text{I.F.}) dx + C \\
 \Rightarrow y \cdot x^2 &= \int(x \log x \cdot x^2) dx + C \\
 \Rightarrow x^2 y &= \int(x^3 \log x) dx + C \\
 \Rightarrow x^2 y &= \log x \cdot \int x^3 dx - \int \left[\frac{d}{dx}(\log x) \cdot \int x^3 dx \right] dx + C \\
 \Rightarrow x^2 y &= \log x \cdot \frac{x^4}{4} - \int \left(\frac{1}{x} \cdot \frac{x^4}{4} \right) dx + C \\
 \Rightarrow x^2 y &= \frac{x^4 \log x}{4} - \frac{1}{4} \int x^3 dx + C \\
 \Rightarrow x^2 y &= \frac{x^4 \log x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C \\
 \Rightarrow x^2 y &= \frac{1}{16} x^4 (4 \log x - 1) + C \\
 \Rightarrow y &= \frac{1}{16} x^2 (4 \log x - 1) + Cx^{-2}
 \end{aligned}$$

Question 7:

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

The given differential equation is:

$$\begin{aligned}
 x \log x \frac{dy}{dx} + y &= \frac{2}{x} \log x \\
 \Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} &= \frac{2}{x^2}
 \end{aligned}$$

This equation is the form of a linear differential equation as:

$$\frac{dy}{dx} + py = Q \quad (\text{where } p = \frac{1}{x \log x} \text{ and } Q = \frac{2}{x^2})$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x.$$

The general solution of the given differential equation is given by the relation,



$$y(\text{I.F.}) = \int(Q \times \text{I.F.}) dx + C$$

$$\Rightarrow y \log x = \int \left(\frac{2}{x^2} \log x \right) dx + C \quad \dots(1)$$

$$\begin{aligned} \text{Now, } \int \left(\frac{2}{x^2} \log x \right) dx &= 2 \int \left(\log x \cdot \frac{1}{x^2} \right) dx \\ &= 2 \left[\log x \cdot \int \frac{1}{x^2} dx - \int \left\{ \frac{d}{dx} (\log x) \cdot \int \frac{1}{x^2} dx \right\} dx \right] \\ &= 2 \left[\log x \left(-\frac{1}{x} \right) - \int \left(\frac{1}{x} \cdot \left(-\frac{1}{x} \right) \right) dx \right] \\ &= 2 \left[-\frac{\log x}{x} + \int \frac{1}{x^2} dx \right] \\ &= 2 \left[-\frac{\log x}{x} - \frac{1}{x} \right] \\ &= -\frac{2}{x} (1 + \log x) \end{aligned}$$

Substituting the value of $\int \left(\frac{2}{x^2} \log x \right) dx$ in equation (1), we get:

$$y \log x = -\frac{2}{x} (1 + \log x) + C$$

This is the required general solution of the given differential equation.

Question 8:

$$(1+x^2) dy + 2xy dx = \cot x dx \quad (x \neq 0)$$

$$(1+x^2) dy + 2xy dx = \cot x dx$$

$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$$

This equation is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \quad (\text{where } p = \frac{2x}{1+x^2} \text{ and } Q = \frac{\cot x}{1+x^2})$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2.$$



The general solution of the given differential equation is given by the relation,

$$\begin{aligned}
 y(\text{I.F.}) &= \int(Q \times \text{I.F.}) dx + C \\
 \Rightarrow y(1+x^2) &= \int \left[\frac{\cot x}{1+x^2} \times (1+x^2) \right] dx + C \\
 \Rightarrow y(1+x^2) &= \int \cot x dx + C \\
 \Rightarrow y(1+x^2) &= \log |\sin x| + C
 \end{aligned}$$

Question 9:

$$x \frac{dy}{dx} + y - x + xy \cot x = 0 \quad (x \neq 0)$$

$$\begin{aligned}
 x \frac{dy}{dx} + y - x + xy \cot x &= 0 \\
 \Rightarrow x \frac{dy}{dx} + y(1+x \cot x) &= x \\
 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x \right) y &= 1
 \end{aligned}$$

This equation is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \quad (\text{where } p = \frac{1}{x} + \cot x \text{ and } Q = 1)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \left(\frac{1}{x} + \cot x \right) dx} = e^{\log x + \log(\sin x)} = e^{\log(x \sin x)} = x \sin x.$$

The general solution of the given differential equation is given by the relation,

$$\begin{aligned}
 y(\text{I.F.}) &= \int(Q \times \text{I.F.}) dx + C \\
 \Rightarrow y(x \sin x) &= \int(1 \times x \sin x) dx + C \\
 \Rightarrow y(x \sin x) &= \int(x \sin x) dx + C \\
 \Rightarrow y(x \sin x) &= x \int \sin x dx - \int \left[\frac{d}{dx}(x) \cdot \int \sin x dx \right] + C \\
 \Rightarrow y(x \sin x) &= x(-\cos x) - \int 1 \cdot (-\cos x) dx + C \\
 \Rightarrow y(x \sin x) &= -x \cos x + \sin x + C \\
 \Rightarrow y &= \frac{-x \cos x}{x \sin x} + \frac{\sin x}{x \sin x} + \frac{C}{x \sin x} \\
 \Rightarrow y &= -\cot x + \frac{1}{x} + \frac{C}{x \sin x}
 \end{aligned}$$

Question 10:



$$(x+y)\frac{dy}{dx}=1$$

$$(x+y)\frac{dy}{dx}=1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$$

$$\Rightarrow \frac{dx}{dy} = x+y$$

$$\Rightarrow \frac{dx}{dy} - x = y$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + px = Q \text{ (where } p = -1 \text{ and } Q = y)$$

$$\text{Now, I.F.} = e^{\int p \, dy} = e^{\int -1 \, dy} = e^{-y}.$$

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int (Q \times \text{I.F.}) \, dy + C$$

$$\Rightarrow xe^{-y} = \int (y \cdot e^{-y}) \, dy + C$$

$$\Rightarrow xe^{-y} = y \cdot \int e^{-y} \, dy - \int \left[\frac{d}{dy}(y) \int e^{-y} \, dy \right] \, dy + C$$

$$\Rightarrow xe^{-y} = y(-e^{-y}) - \int (-e^{-y}) \, dy + C$$

$$\Rightarrow xe^{-y} = -ye^{-y} + \int e^{-y} \, dy + C$$

$$\Rightarrow xe^{-y} = -ye^{-y} - e^{-y} + C$$

$$\Rightarrow x = -y - 1 + Ce^y$$

$$\Rightarrow x + y + 1 = Ce^y$$

Question 11:

$$y \, dx + (x - y^2) \, dy = 0$$



$$y dx + (x - y^2) dy = 0$$

$$\Rightarrow y dx = (y^2 - x) dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{y^2 - x}{y} = y - \frac{x}{y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y} = y$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + px = Q \text{ (where } p = \frac{1}{y} \text{ and } Q = y)$$

$$\text{Now, I.F.} = e^{\int p dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y.$$

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int (Q \times \text{I.F.}) dy + C$$

$$\Rightarrow xy = \int (y \cdot y) dy + C$$

$$\Rightarrow xy = \int y^2 dy + C$$

$$\Rightarrow xy = \frac{y^3}{3} + C$$

$$\Rightarrow x = \frac{y^2}{3} + \frac{C}{y}$$

Question 12:

$$(x + 3y^2) \frac{dy}{dx} = y \quad (y > 0)$$

$$(x + 3y^2) \frac{dy}{dx} = y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x + 3y^2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x + 3y^2}{y} = \frac{x}{y} + 3y$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$$



This is a linear differential equation of the form:

$$\frac{dx}{dy} + px = Q \text{ (where } p = -\frac{1}{y} \text{ and } Q = 3y)$$

$$\text{Now, I.F.} = e^{\int p dy} = e^{-\int \frac{dy}{y}} = e^{-\log y} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}.$$

The general solution of the given differential equation is given by the relation,

$$x(\text{I.F.}) = \int(Q \times \text{I.F.}) dy + C$$

$$\Rightarrow x \times \frac{1}{y} = \int \left(3y \times \frac{1}{y} \right) dy + C$$

$$\Rightarrow \frac{x}{y} = 3y + C$$

$$\Rightarrow x = 3y^2 + Cy$$

Question 13:

$$\frac{dy}{dx} + 2y \tan x = \sin x; y = 0 \text{ when } x = \frac{\pi}{3}$$

The given differential equation is $\frac{dy}{dx} + 2y \tan x = \sin x$.

This is a linear equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = 2 \tan x \text{ and } Q = \sin x)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int 2 \tan x dx} = e^{2 \log |\sec x|} = e^{\log(\sec^2 x)} = \sec^2 x.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int(Q \times \text{I.F.}) dx + C$$

$$\Rightarrow y(\sec^2 x) = \int(\sin x \cdot \sec^2 x) dx + C$$

$$\Rightarrow y \sec^2 x = \int(\sec x \cdot \tan x) dx + C$$

$$\Rightarrow y \sec^2 x = \sec x + C \quad \dots(1)$$



Now, $y = 0$ at $x = \frac{\pi}{3}$.

Therefore,

$$0 \times \sec^2 \frac{\pi}{3} = \sec \frac{\pi}{3} + C$$

$$\Rightarrow 0 = 2 + C$$

$$\Rightarrow C = -2$$

Substituting $C = -2$ in equation (1), we get:

$$y \sec^2 x = \sec x - 2$$

$$\Rightarrow y = \cos x - 2 \cos^2 x$$

Hence, the required solution of the given differential equation is $y = \cos x - 2 \cos^2 x$.

Question 14:

$$(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0 \text{ when } x = 1$$

$$(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = \frac{2x}{1+x^2} \text{ and } Q = \frac{1}{(1+x^2)^2} \text{)}$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int \frac{2x dx}{1+x^2}} = e^{\log(1+x^2)} = 1+x^2.$$

The general solution of the given differential equation is given by the relation,



$$y(\text{I.F.}) = \int(Q \times \text{I.F.}) dx + C$$

$$\Rightarrow y(1+x^2) = \int \left[\frac{1}{(1+x^2)^2} \cdot (1+x^2) \right] dx + C$$

$$\Rightarrow y(1+x^2) = \int \frac{1}{1+x^2} dx + C$$

$$\Rightarrow y(1+x^2) = \tan^{-1} x + C \quad \dots(1)$$

Now, $y = 0$ at $x = 1$.

Therefore,

$$0 = \tan^{-1} 1 + C$$

$$\Rightarrow C = -\frac{\pi}{4}$$

Substituting $C = -\frac{\pi}{4}$ in equation (1), we get:

$$y(1+x^2) = \tan^{-1} x - \frac{\pi}{4}$$

This is the required general solution of the given differential equation.

Question 15:

$$\frac{dy}{dx} - 3y \cot x = \sin 2x; y = 2 \text{ when } x = \frac{\pi}{2}$$

The given differential equation is $\frac{dy}{dx} - 3y \cot x = \sin 2x$.

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = -3 \cot x \text{ and } Q = \sin 2x)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{-3 \int \cot x dx} = e^{-3 \log |\sin x|} = e^{\log \left| \frac{1}{\sin^3 x} \right|} = \frac{1}{\sin^3 x}$$

The general solution of the given differential equation is given by the relation,



$$y(\text{I.F.}) = \int (Q \times \text{I.F.}) dx + C$$

$$\Rightarrow y \cdot \frac{1}{\sin^3 x} = \int \left[\sin 2x \cdot \frac{1}{\sin^3 x} \right] dx + C$$

$$\Rightarrow y \operatorname{cosec}^3 x = 2 \int (\cot x \operatorname{cosec} x) dx + C$$

$$\Rightarrow y \operatorname{cosec}^3 x = 2 \operatorname{cosec} x + C$$

$$\Rightarrow y = -\frac{2}{\operatorname{cosec}^2 x} + \frac{3}{\operatorname{cosec}^3 x}$$

$$\Rightarrow y = -2 \sin^2 x + C \sin^3 x \quad \dots(1)$$

Now, $y = 2$ at $x = \frac{\pi}{2}$.

Therefore, we get:

$$2 = -2 + C$$

$$\Rightarrow C = 4$$

Substituting $C = 4$ in equation (1), we get:

$$y = -2 \sin^2 x + 4 \sin^3 x$$

$$\Rightarrow y = 4 \sin^3 x - 2 \sin^2 x$$

This is the required particular solution of the given differential equation.

Question 16:

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

Let $F(x, y)$ be the curve passing through the origin.

At point (x, y) , the slope of the curve will be $\frac{dy}{dx}$.

According to the given information:



$$\frac{dy}{dx} = x + y$$

$$\Rightarrow \frac{dy}{dx} - y = x$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = -1 \text{ and } Q = x)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int (-1) dx} = e^{-x}.$$

The general solution of the given differential equation is given by the relation,

$$y(\text{I.F.}) = \int (Q \times \text{I.F.}) dx + C$$

$$\Rightarrow ye^{-x} = \int xe^{-x} dx + C \quad \dots(1)$$

$$\text{Now, } \int xe^{-x} dx = x \int e^{-x} dx - \int \left[\frac{d}{dx}(x) \cdot \int e^{-x} dx \right] dx.$$

$$= -xe^{-x} - \int -e^{-x} dx$$

$$= -xe^{-x} + (-e^{-x})$$

$$= -e^{-x}(x+1)$$

Substituting in equation (1), we get:

$$ye^{-x} = -e^{-x}(x+1) + C$$

$$\Rightarrow y = -(x+1) + Ce^x$$

$$\Rightarrow x + y + 1 = Ce^x \quad \dots(2)$$

The curve passes through the origin.

Therefore, equation (2) becomes:

$$1 = C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (2), we get:



$$x + y + 1 = e^x$$

Hence, the required equation of curve passing through the origin is $x + y + 1 = e^x$.

Question 17:

Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Let $F(x, y)$ be the curve and let (x, y) be a point on the curve. The slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given information:

$$\begin{aligned} \frac{dy}{dx} + 5 &= x + y \\ \Rightarrow \frac{dy}{dx} - y &= x - 5 \end{aligned}$$

This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = -1 \text{ and } Q = x - 5)$$

$$\text{Now, I.F.} = e^{\int p dx} = e^{\int (-1) dx} = e^{-x}.$$

The general equation of the curve is given by the relation,

$$\begin{aligned} y(\text{I.F.}) &= \int (Q \times \text{I.F.}) dx + C \\ \Rightarrow y \cdot e^{-x} &= \int (x - 5)e^{-x} dx + C \quad \dots(1) \end{aligned}$$

$$\begin{aligned} \text{Now, } \int (x - 5)e^{-x} dx &= (x - 5) \int e^{-x} dx - \int \left[\frac{d}{dx} (x - 5) \cdot \int e^{-x} dx \right] dx \\ &= (x - 5)(-e^{-x}) - \int (-e^{-x}) dx \\ &= (5 - x)e^{-x} + (-e^{-x}) \\ &= (4 - x)e^{-x} \end{aligned}$$

Therefore, equation (1) becomes:



$$ye^{-x} = (4-x)e^{-x} + C$$

$$\Rightarrow y = 4 - x + Ce^x$$

$$\Rightarrow x + y - 4 = Ce^x \quad \dots(2)$$

The curve passes through point (0, 2).

Therefore, equation (2) becomes:

$$0 + 2 - 4 = Ce^0$$

$$\Rightarrow -2 = C$$

$$\Rightarrow C = -2$$

Substituting $C = -2$ in equation (2), we get:

$$x + y - 4 = -2e^x$$

$$\Rightarrow y = 4 - x - 2e^x$$

This is the required equation of the curve.

Question 18:

The integrating factor of the differential equation $x \frac{dy}{dx} - y = 2x^2$ is

A. e^{-x}

B. e^{-y}

C. $\frac{1}{x}$

D. x

The given differential equation is:

$$x \frac{dy}{dx} - y = 2x^2$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$$



This is a linear differential equation of the form:

$$\frac{dy}{dx} + py = Q \text{ (where } p = -\frac{1}{x} \text{ and } Q = 2x)$$

The integrating factor (I.F) is given by the relation,

$$e^{\int p dx}$$

$$\therefore \text{I.F} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$$

Hence, the correct answer is C.

Question 19:

The integrating factor of the differential equation.

$$(1-y^2) \frac{dx}{dy} + yx = ay \text{ } (-1 < y < 1) \text{ is}$$

A. $\frac{1}{y^2 - 1}$

B. $\frac{1}{\sqrt{y^2 - 1}}$

C. $\frac{1}{1 - y^2}$

D. $\frac{1}{\sqrt{1 - y^2}}$

The given differential equation is:

$$(1-y^2) \frac{dx}{dy} + yx = ay$$

$$\Rightarrow \frac{dy}{dx} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$$



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This is a linear differential equation of the form:

$$\frac{dx}{dy} + py = Q \text{ (where } p = \frac{y}{1-y^2} \text{ and } Q = \frac{ay}{1-y^2}\text{)}$$

The integrating factor (I.F) is given by the relation,

$$e^{\int p dx}$$

$$\therefore \text{I.F} = e^{\int p dy} = e^{\int \frac{y}{1-y^2} dy} = e^{-\frac{1}{2} \log(1-y^2)} = e^{\log \left[\frac{1}{\sqrt{1-y^2}} \right]} = \frac{1}{\sqrt{1-y^2}}$$

Hence, the correct answer is D.