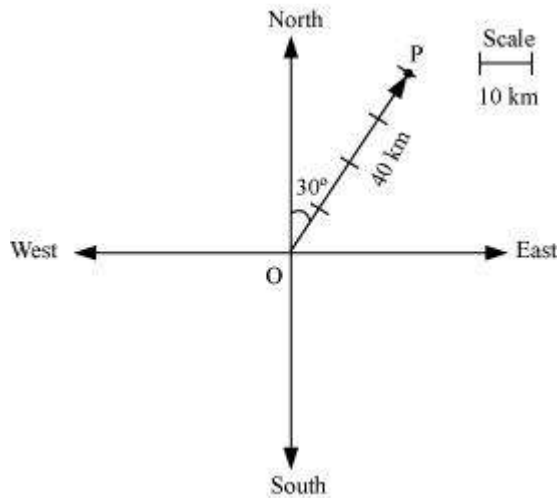




## Exercise-10.1

## Question 1:

Represent graphically a displacement of 40 km,  $30^\circ$  east of north.



Here, vector  $\overline{OP}$  represents the displacement of 40 km,  $30^\circ$  East of North.

## Question 2:

Classify the following measures as scalars and vectors.

(i) 10 kg (ii) 2 metres north-west (iii)  $40^\circ$

(iv) 40 watt (v)  $10^{-19}$  coulomb (vi)  $20 \text{ m/s}^2$

(i) 10 kg is a scalar quantity because it involves only magnitude.

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.

(iii)  $40^\circ$  is a scalar quantity as it involves only magnitude.

(iv) 40 watts is a scalar quantity as it involves only magnitude.

(v)  $10^{-19}$  coulomb is a scalar quantity as it involves only magnitude.

(vi)  $20 \text{ m/s}^2$  is a vector quantity as it involves magnitude as well as direction.



Question 3:

Classify the following as scalar and vector quantities.

(i) time period (ii) distance (iii) force

(iv) velocity (v) work done

(i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

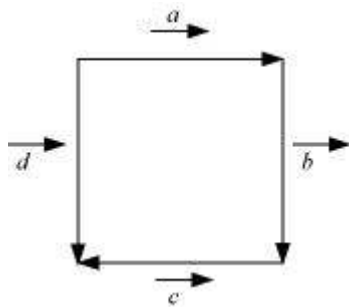
(iii) Force is a vector quantity as it involves both magnitude and direction.

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.

(v) Work done is a scalar quantity as it involves only magnitude.

Question 4:

In Figure, identify the following vectors.



(i) Coinitial (ii) Equal (iii) Collinear but not equal

(i) Vectors  $\vec{a}$  and  $\vec{d}$  are coinitial because they have the same initial point.

(ii) Vectors  $\vec{b}$  and  $\vec{d}$  are equal because they have the same magnitude and direction.



(iii) Vectors  $\vec{a}$  and  $\vec{c}$  are collinear but not equal. This is because although they are parallel, their directions are not the same.

Question 5:

Answer the following as true or false.

- (i)  $\vec{a}$  and  $-\vec{a}$  are collinear.
- (ii) Two collinear vectors are always equal in magnitude.
- (iii) Two vectors having same magnitude are collinear.
- (iv) Two collinear vectors having the same magnitude are equal.

(i) True.

Vectors  $\vec{a}$  and  $-\vec{a}$  are parallel to the same line.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

(iii) False.

It is not necessary for two vectors having the same magnitude to be parallel to the same line.

(iv) False.

Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

Exercise-10.2

Question 1:

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$



The given vectors are:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$$

$$= \sqrt{4 + 49 + 9}$$

$$= \sqrt{62}$$

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$$

Question 2:

Write two different vectors having same magnitude.

Consider  $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$  and  $\vec{b} = (2\hat{i} + \hat{j} - 3\hat{k})$ .

It can be observed that  $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$  and

$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$ .

Hence,  $\vec{a}$  and  $\vec{b}$  are two different vectors having the same magnitude. The vectors are different because they have different directions.

Question 3:

Write two different vectors having same direction.

Consider  $\vec{p} = (\hat{i} + \hat{j} + \hat{k})$  and  $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$ .

The direction cosines of  $\vec{p}$  are given by,

$$l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad \text{and } n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of  $\vec{q}$  are given by

$$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \quad m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{and } n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of  $\vec{p}$  and  $\vec{q}$  are the same. Hence, the two vectors have the same direction.



Question 4:

Find the values of  $x$  and  $y$  so that the vectors  $2\hat{i} + 3\hat{j}$  and  $x\hat{i} + y\hat{j}$  are equal

The two vectors  $2\hat{i} + 3\hat{j}$  and  $x\hat{i} + y\hat{j}$  will be equal if their corresponding components are equal.

Hence, the required values of  $x$  and  $y$  are 2 and 3 respectively.

Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

The vector with the initial point P (2, 1) and terminal point Q (-5, 7) can be given by,

$$\begin{aligned}\overline{PQ} &= (-5 - 2)\hat{i} + (7 - 1)\hat{j} \\ \Rightarrow \overline{PQ} &= -7\hat{i} + 6\hat{j}\end{aligned}$$

Hence, the required scalar components are  $-7$  and  $6$  while the vector components are  $-7\hat{i}$  and  $6\hat{j}$ .

Question 6:

Find the sum of the vectors  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ .

The given vectors are  $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ ,  $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$  and  $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$ .

$$\begin{aligned}\therefore \vec{a} + \vec{b} + \vec{c} &= (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k} \\ &= 0\hat{i} - 4\hat{j} - 1\hat{k} \\ &= -4\hat{j} - \hat{k}\end{aligned}$$

Question 7:

Find the unit vector in the direction of the vector  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ .



The unit vector  $\hat{a}$  in the direction of vector  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  is given by  $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ .

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

Question 8:

Find the unit vector in the direction of vector  $\overline{PQ}$ , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overline{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$|\overline{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of  $\overline{PQ}$  is

$$\frac{\overline{PQ}}{|\overline{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Question 9:

For given vectors,  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ , find the unit vector in the direction of the vector  $\vec{a} + \vec{b}$

The given vectors are  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  and  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ .

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

$$\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$



Hence, the unit vector in the direction of  $(\vec{a} + \vec{b})$  is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

Question 10:

Find a vector in the direction of vector  $5\hat{i} - \hat{j} + 2\hat{k}$  which has magnitude 8 units.

$$\text{Let } \vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}.$$

$$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector  $5\hat{i} - \hat{j} + 2\hat{k}$  which has magnitude 8 units is given by,

$$8\hat{a} = 8 \left( \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} \right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8 \left( \frac{5\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{30}} \right)$$

$$= \frac{40}{\sqrt{30}}\vec{i} - \frac{8}{\sqrt{30}}\vec{j} + \frac{16}{\sqrt{30}}\vec{k}$$

Question 11:

Show that the vectors  $2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $-4\hat{i} + 6\hat{j} - 8\hat{k}$  are collinear.

$$\text{Let } \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ and } \vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}.$$

$$\text{It is observed that } \vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$$

$$\therefore \vec{b} = \lambda\vec{a}$$

where,

$$\lambda = -2$$



Hence, the given vectors are collinear.

Question 12:

Find the direction cosines of the vector  $\hat{i} + 2\hat{j} + 3\hat{k}$

$$\text{Let } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}.$$

$$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

Hence, the direction cosines of  $\vec{a}$  are  $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ .

Question 13:

Find the direction cosines of the vector joining the points A (1, 2, -3) and

B (-1, -2, 1) directed from A to B.

The given points are A (1, 2, -3) and B (-1, -2, 1).

$$\therefore \vec{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$$

$$\Rightarrow \vec{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\vec{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

Hence, the direction cosines of  $\vec{AB}$  are  $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ .

Question 14:

Show that the vector  $\hat{i} + \hat{j} + \hat{k}$  is equally inclined to the axes OX, OY, and OZ.

$$\text{Let } \vec{a} = \hat{i} + \hat{j} + \hat{k}.$$

Then,

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Therefore, the direction cosines of  $\vec{a}$  are  $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ .





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Now, let  $\alpha$ ,  $\beta$ , and  $\gamma$  be the angles formed by  $\vec{a}$  with the positive directions of  $x$ ,  $y$ , and  $z$  axes.

Then, we have  $\cos \alpha = \frac{1}{\sqrt{3}}$ ,  $\cos \beta = \frac{1}{\sqrt{3}}$ ,  $\cos \gamma = \frac{1}{\sqrt{3}}$ .

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

### Question 15:

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are  $\hat{i} + 2\hat{j} - \hat{k}$  and  $-\hat{i} + \hat{j} + \hat{k}$  respectively, in the ratio 2:1

(i) internally

(ii) externally

The position vector of point R dividing the line segment joining two points

P and Q in the ratio  $m:n$  is given by:

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m+n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m-n}$$

Position vectors of P and Q are given as:

$$\overline{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overline{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,



$$\begin{aligned}\overline{\text{OR}} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})}{3} \\ &= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}\end{aligned}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\begin{aligned}\overline{\text{OR}} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2-1} = (-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) \\ &= -3\hat{i} + 3\hat{k}\end{aligned}$$

Question 16:

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, -2).

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, -2) is given by,

$$\begin{aligned}\overline{\text{OR}} &= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2} \\ &= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}\end{aligned}$$

Question 17:

Show that the points A, B and C with position vectors,  $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ , respectively form the vertices of a right angled triangle.

Position vectors of points A, B, and C are respectively given as:

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$



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$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \overline{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overline{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\overline{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1+9+25 = 35$$

$$|\overline{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1+4+36 = 41$$

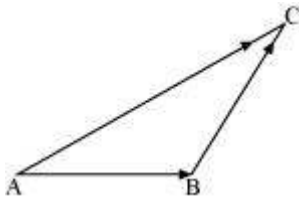
$$|\overline{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4+1+1 = 6$$

$$\therefore |\overline{AB}|^2 + |\overline{CA}|^2 = 35 + 6 = 41 = |\overline{BC}|^2$$

Hence, ABC is a right-angled triangle.

Question 18:

In triangle ABC which of the following is **not** true:

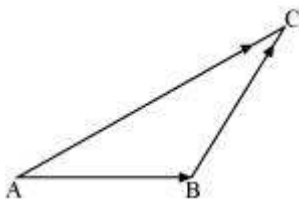


A.  $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

B.  $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$

C.  $\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$

D.  $\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$





On applying the triangle law of addition in the given triangle, we have:

$$\overline{AB} + \overline{BC} = \overline{AC} \quad \dots(1)$$

$$\Rightarrow \overline{AB} + \overline{BC} = -\overline{CA}$$

$$\Rightarrow \overline{AB} + \overline{BC} + \overline{CA} = \vec{0} \quad \dots(2)$$

∴ The equation given in alternative A is true.

$$\overline{AB} + \overline{BC} = \overline{AC}$$

$$\Rightarrow \overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$$

∴ The equation given in alternative B is true.

From equation (2), we have:

$$\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$$

∴ The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$$\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$$

$$\Rightarrow \overline{AB} + \overline{BC} = \overline{CA} \quad \dots(3)$$

From equations (1) and (3), we have:

$$\overline{AC} = \overline{CA}$$

$$\Rightarrow \overline{AC} = -\overline{AC}$$

$$\Rightarrow \overline{AC} + \overline{AC} = \vec{0}$$

$$\Rightarrow 2\overline{AC} = \vec{0}$$

$$\Rightarrow \overline{AC} = \vec{0}, \text{ which is not true.}$$

Hence, the equation given in alternative C is **incorrect**.

The correct answer is C.

Question 19:

If  $\vec{a}$  and  $\vec{b}$  are two collinear vectors, then which of the following are **incorrect**:

A.  $\vec{b} = \lambda\vec{a}$ , for some scalar  $\lambda$

B.  $\vec{a} = \pm\vec{b}$



C. the respective components of  $\vec{a}$  and  $\vec{b}$  are proportional

D. both the vectors  $\vec{a}$  and  $\vec{b}$  have same direction, but different magnitudes

If  $\vec{a}$  and  $\vec{b}$  are two collinear vectors, then they are parallel.

Therefore, we have:

$$\vec{b} = \lambda \vec{a} \text{ (For some scalar } \lambda)$$

If  $\lambda = \pm 1$ , then  $\vec{a} = \pm \vec{b}$ .

If  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ , then

$$\vec{b} = \lambda \vec{a}.$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Thus, the respective components of  $\vec{a}$  and  $\vec{b}$  are proportional.

However, vectors  $\vec{a}$  and  $\vec{b}$  can have different directions.

Hence, the statement given in **D** is **incorrect**.

The correct answer is **D**.

Exercise-10.3

Question 1:

Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes  $\sqrt{3}$  and 2, respectively having  $\vec{a} \cdot \vec{b} = \sqrt{6}$ .

It is given that,



$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and } \vec{a} \cdot \vec{b} = \sqrt{6}$$

Now, we know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the given vectors  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$ .

Question 2:

Find the angle between the vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$

The given vectors are  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ .

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$\begin{aligned} \text{Now, } \vec{a} \cdot \vec{b} &= (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k}) \\ &= 1 \cdot 3 + (-2)(-2) + 3 \cdot 1 \\ &= 3 + 4 + 3 \\ &= 10 \end{aligned}$$

Also, we know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{10}{14}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$$



Question 3:

Find the projection of the vector  $\hat{i} - \hat{j}$  on the vector  $\hat{i} + \hat{j}$ .

Let  $\vec{a} = \hat{i} - \hat{j}$  and  $\vec{b} = \hat{i} + \hat{j}$ .

Now, projection of vector  $\vec{a}$  on  $\vec{b}$  is given by,

$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{1+1}} \{1 \cdot 1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1-1) = 0$$

Hence, the projection of vector  $\vec{a}$  on  $\vec{b}$  is 0.

Question 4:

Find the projection of the vector  $\hat{i} + 3\hat{j} + 7\hat{k}$  on the vector  $7\hat{i} - \hat{j} + 8\hat{k}$ .

Let  $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$  and  $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ .

Now, projection of vector  $\vec{a}$  on  $\vec{b}$  is given by,

$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}} \{1(7) + 3(-1) + 7(8)\} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$$

Question 5:

Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Also, show that they are mutually perpendicular to each other.

$$\text{Let } \vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k},$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}.$$



$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a} \cdot \vec{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(\frac{-6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$

$$\vec{b} \cdot \vec{c} = \frac{3}{7} \times \frac{6}{7} + \left(\frac{-6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(\frac{-3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$$

$$\vec{c} \cdot \vec{a} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(\frac{-3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$$

Hence, the given three vectors are mutually perpendicular to each other.

Question 6:

Find  $|\vec{a}|$  and  $|\vec{b}|$ , if  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$  and  $|\vec{a}| = 8|\vec{b}|$ .

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8 \quad [|\vec{a}| = 8|\vec{b}|]$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} \quad [\text{Magnitude of a vector is non-negative}]$$

$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$





$$|\vec{a}| = 8|\vec{b}| = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

Question 7:

Evaluate the product  $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$ .

$$\begin{aligned} & (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) \\ &= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b} \\ &= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b} \\ &= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2 \end{aligned}$$

Question 8:

Find the magnitude of two vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude and such that the angle between them is  $60^\circ$  and their scalar product is  $\frac{1}{2}$ .

Let  $\theta$  be the angle between the vectors  $\vec{a}$  and  $\vec{b}$ .

It is given that  $|\vec{a}| = |\vec{b}|$ ,  $\vec{a} \cdot \vec{b} = \frac{1}{2}$ , and  $\theta = 60^\circ$ . ... (1)

We know that  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ .

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Question 9:

Find  $|\vec{x}|$ , if for a unit vector  $\vec{a}$ ,  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$ .



$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

$$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$$

$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$$

$$\Rightarrow |\vec{x}|^2 - 1 = 12 \quad [|\vec{a}| = 1 \text{ as } \vec{a} \text{ is a unit vector}]$$

$$\Rightarrow |\vec{x}|^2 = 13$$

$$\therefore |\vec{x}| = \sqrt{13}$$

Question 10:

If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$ , then find the value of  $\lambda$ .

The given vectors are  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ , and  $\vec{c} = 3\hat{i} + \hat{j}$ .

Now,

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If  $(\vec{a} + \lambda\vec{b})$  is perpendicular to  $\vec{c}$ , then

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0.$$

$$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of  $\lambda$  is 8.

Question 11:

Show that  $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$  is perpendicular to  $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ , for any two nonzero vectors  $\vec{a}$  and  $\vec{b}$

$$(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a})$$

$$= |\vec{a}|^2 \vec{b} \cdot \vec{b} - |\vec{a}|\vec{b}|\vec{b} \cdot \vec{a} + |\vec{b}|\vec{a}|\vec{a} \cdot \vec{b} - |\vec{b}|^2 \vec{a} \cdot \vec{a}$$

$$= |\vec{a}|^2 |\vec{b}|^2 - |\vec{b}|^2 |\vec{a}|^2$$

$$= 0$$



Hence,  $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$  and  $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$  are perpendicular to each other.

Question 12:

If  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$ , then what can be concluded about the vector  $\vec{b}$  ?

It is given that  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$ .

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \vec{a}$  is a zero vector.

Hence, vector  $\vec{b}$  satisfying  $\vec{a} \cdot \vec{b} = 0$  can be any vector.

Question 14:

If either vector  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \cdot \vec{b} = 0$ . But the converse need not be true. Justify your answer with an example.

Consider  $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$ .

Then,

$$\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Question 15:

If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively, then find  $\angle ABC$ . [ $\angle ABC$  is the angle between the vectors  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$ ]

The vertices of  $\Delta ABC$  are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2).



Also, it is given that  $\angle ABC$  is the angle between the vectors  $\overline{BA}$  and  $\overline{BC}$ .

$$\overline{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \overline{BA} \cdot \overline{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$$

$$|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$$

$$|\overline{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$$

Now, it is known that:

$$\overline{BA} \cdot \overline{BC} = |\overline{BA}| |\overline{BC}| \cos(\angle ABC)$$

$$\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$$

$$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

Question 16:

Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, -1) are collinear.

The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, -1).

$$\therefore \overline{AB} = (2 - 1)\hat{i} + (6 - 2)\hat{j} + (3 - 7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{BC} = (3 - 2)\hat{i} + (10 - 6)\hat{j} + (-1 - 3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{AC} = (3 - 1)\hat{i} + (10 - 2)\hat{j} + (-1 - 7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$|\overline{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$$

$$\therefore |\overline{AC}| = |\overline{AB}| + |\overline{BC}|$$

Hence, the given points A, B, and C are collinear.



Question 17:

Show that the vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  form the vertices of a right angled triangle.

Let vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  be position vectors of points A, B, and C respectively.

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k} \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

Now, vectors  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  represent the sides of  $\Delta ABC$ .

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k}, \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\therefore \overline{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overline{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$|\overline{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$$

$$|\overline{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|\overline{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$\therefore |\overline{BC}|^2 + |\overline{AC}|^2 = 6 + 35 = 41 = |\overline{AB}|^2$$

Hence,  $\Delta ABC$  is a right-angled triangle.

Question 18:

If  $\vec{a}$  is a nonzero vector of magnitude 'a' and  $\lambda$  a nonzero scalar, then  $\lambda \vec{a}$  is unit vector if

(A)  $\lambda = 1$  (B)  $\lambda = -1$  (C)  $a = |\lambda|$

(D)  $a = \frac{1}{|\lambda|}$

Vector  $\lambda \vec{a}$  is a unit vector if  $|\lambda \vec{a}| = 1$ .



Now,

$$|\lambda \vec{a}| = 1$$

$$\Rightarrow |\lambda| |\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \quad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \quad [|\vec{a}| = a]$$

Hence, vector  $\lambda \vec{a}$  is a unit vector if  $a = \frac{1}{|\lambda|}$ .

The correct answer is D.

### Exercise-10.4

Question 1:

Find  $|\vec{a} \times \vec{b}|$ , if  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

We have,

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

Question 2:

Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ .

We have,



$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \quad \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\begin{aligned} \therefore |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| &= \sqrt{16^2 + (-16)^2 + (-8)^2} \\ &= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2} \\ &= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24 \end{aligned}$$

Hence, the unit vector perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  is given by the relation,

$$\begin{aligned} &= \pm \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|} = \pm \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24} \\ &= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k} \end{aligned}$$

Question 3:

If a unit vector  $\vec{a}$  makes an angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$ , then find  $\theta$  and hence, the compounds of  $\vec{a}$ .

Let unit vector  $\vec{a}$  have  $(a_1, a_2, a_3)$  components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since  $\vec{a}$  is a unit vector,  $|\vec{a}| = 1$ .

Also, it is given that  $\vec{a}$  makes angles  $\frac{\pi}{3}$  with  $\hat{i}$ ,  $\frac{\pi}{4}$  with  $\hat{j}$ , and an acute angle  $\theta$  with  $\hat{k}$ .

Then, we have:



$$\cos \frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_1 \quad [|\vec{a}| = 1]$$

$$\cos \frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \quad [|\vec{a}| = 1]$$

$$\text{Also, } \cos \theta = \frac{a_3}{|\vec{a}|}$$

$$\Rightarrow a_3 = \cos \theta$$

Now,

$$|\vec{a}| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

Hence,  $\theta = \frac{\pi}{3}$  and the components of  $\vec{a}$  are  $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ .

Question 4:

Show that

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$$





$$\begin{aligned}
 & (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) \\
 &= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b} && \text{[By distributivity of vector product over addition]} \\
 &= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b} && \text{[Again, by distributivity of vector product over addition]} \\
 &= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0} \\
 &= 2\vec{a} \times \vec{b}
 \end{aligned}$$

Question 5:

Find  $\lambda$  and  $\mu$  if  $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$ .

$$\begin{aligned}
 & (2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0} \\
 & \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k} \\
 & \Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}
 \end{aligned}$$

On comparing the corresponding components, we have:

$$\begin{aligned}
 6\mu - 27\lambda &= 0 \\
 2\mu - 27 &= 0 \\
 2\lambda - 6 &= 0
 \end{aligned}$$

Now,

$$2\lambda - 6 = 0 \Rightarrow \lambda = 3$$

$$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$$

Hence,  $\lambda = 3$  and  $\mu = \frac{27}{2}$ .

Question 6:

Given that  $\vec{a} \cdot \vec{b} = 0$  and  $\vec{a} \times \vec{b} = \vec{0}$ . What can you conclude about the vectors  $\vec{a}$  and  $\vec{b}$ ?

$$\vec{a} \cdot \vec{b} = 0$$



Then,

(i) Either  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ , or  $\vec{a} \perp \vec{b}$  (in case  $\vec{a}$  and  $\vec{b}$  are non-zero)

$$\vec{a} \times \vec{b} = 0$$

(ii) Either  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ , or  $\vec{a} \parallel \vec{b}$  (in case  $\vec{a}$  and  $\vec{b}$  are non-zero)

But,  $\vec{a}$  and  $\vec{b}$  cannot be perpendicular and parallel simultaneously.

Hence,  $|\vec{a}| = 0$  or  $|\vec{b}| = 0$ .

Question 7:

Let the vectors  $\vec{a}, \vec{b}, \vec{c}$  given as  $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ . Then show that  $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$\text{Now, } \vec{a} \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix}$$

$$= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{j}[a_1(b_3 + c_3) - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)]$$

$$= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \dots(1)$$



$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}[a_2b_3 - a_3b_2] + \hat{j}[b_1a_3 - a_1b_3] + \hat{k}[a_1b_2 - a_2b_1] \quad (2)$$

$$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \hat{i}[a_2c_3 - a_3c_2] + \hat{j}[a_3c_1 - a_1c_3] + \hat{k}[a_1c_2 - a_2c_1] \quad (3)$$

On adding (2) and (3), we get:

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3]$$

$$+ \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad (4)$$

Now, from (1) and (4), we have:

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Hence, the given result is proved.

Question 8:

If either  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \times \vec{b} = \vec{0}$ . Is the converse true? Justify your answer with an example.

Take any parallel non-zero vectors so that  $\vec{a} \times \vec{b} = \vec{0}$ .

$$\text{Let } \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}.$$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:



$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Question 9:

Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

The vertices of triangle ABC are given as A (1, 1, 2), B (2, 3, 5), and C (1, 5, 5).

The adjacent sides  $\overline{AB}$  and  $\overline{BC}$  of  $\Delta ABC$  are given as:

$$\overline{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} |\overline{AB} \times \overline{BC}|$$

$$\overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\therefore |\overline{AB} \times \overline{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$$

Hence, the area of  $\Delta ABC$  is  $\frac{\sqrt{61}}{2}$  square units.

Question 10:

Find the area of the parallelogram whose adjacent sides are determined by the vector  $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$ .



The area of the parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$ .

Adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Hence, the area of the given parallelogram is  $15\sqrt{2}$  square units.

Question 11:

Let the vectors  $\vec{a}$  and  $\vec{b}$  be such that  $|\vec{a}| = 3$  and  $|\vec{b}| = \frac{\sqrt{2}}{3}$ , then  $\vec{a} \times \vec{b}$  is a unit vector, if the angle between  $\vec{a}$  and  $\vec{b}$  is

(A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{2}$

It is given that  $|\vec{a}| = 3$  and  $|\vec{b}| = \frac{\sqrt{2}}{3}$ .

We know that  $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$ , where  $\hat{n}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ .

Now,  $\vec{a} \times \vec{b}$  is a unit vector if  $|\vec{a} \times \vec{b}| = 1$ .



$$|\vec{a} \times \vec{b}| = 1$$

$$\Rightarrow \|\vec{a}\|\|\vec{b}\|\sin\theta\hat{n} = 1$$

$$\Rightarrow \|\vec{a}\|\|\vec{b}\|\sin\theta = 1$$

$$\Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence,  $\vec{a} \times \vec{b}$  is a unit vector if the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$ .

The correct answer is B.

Question 12:

Area of a rectangle having vertices A, B, C, and D with position vectors

$-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$ ,  $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$ ,  $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$  and  $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$  respectively is

(A)  $\frac{1}{2}$  (B) 1

(C) 2 (D) 4

The position vectors of vertices A, B, C, and D of rectangle ABCD are given as:

$$\overline{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \overline{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \overline{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \overline{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

The adjacent sides  $\overline{AB}$  and  $\overline{BC}$  of the given rectangle are given as:



$$\overline{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = 2\hat{i}$$

$$\overline{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\therefore \overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$|\overline{AB} \times \overline{BC}| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$ .

Hence, the area of the given rectangle is  $|\overline{AB} \times \overline{BC}| = 2$  square units.

The correct answer is C.