Chapter-2 Relations and Functions

Miscellaneous

Question 1:

The relation f is defined by $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

The relation g is defined by $g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$

Show that f is a function and g is not a function.

Answer :

The relation f is defined as $f(x)=\begin{cases}x^2, & 0\le x\le 3\\ 3x, & 3\le x\le 10\end{cases}$

It is observed that for

$0 \le x < 3, f(x) = x^2$

$3 < x \le 10, f(x) = 3x$

Also, at $x = 3, f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3, f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation g is defined as $g(x)=\begin{cases}x^2, & 0\le x\le 2\\ 3x, & 2\le x\le 10\end{cases}$

It can be observed that for $x = 2, g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Question 2:

If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Answer :

$$f(x)=x^2$$
$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

Question 3:

Find the domain of the function $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$

Answer :

The given function is $f(x)=\dfrac{x^2+2x+1}{x^2-8x+12}$.

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function f is defined for all real numbers except at $x = 6$ and $x = 2$.

Hence, the domain of f is **R** – {2, 6}.

Question 4:

Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$.

Answer :

The given real function is $f(x)=\sqrt{x-1}$.

It can be seen that $\sqrt{x-1}$ is defined for $(x-1) \geq 0$.

i.e., $f(x)=\sqrt{(x-1)}$ is defined for $x \geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1, \infty)$.

As $x \geq 1 \Rightarrow (x-1) \geq 0 \Rightarrow \sqrt{x-1} \geq 0$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f = [0, \infty)$.

Question 5:

Find the domain and the range of the real function f defined by $f(x) = |x-1|$.

Answer :

The given real function is $f(x) = |x-1|$.

It is clear that $|x-1|$ is defined for all real numbers.

$\therefore$Domain of $f = \mathbf{R}$

Also, for $x \in \mathbf{R}$, $|x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Question 6:

Let $f=\left\{\left(x, \frac{x^2}{1+x^2}\right): x \in \mathbf{R}\right\}$ be a function from **R** into **R**. Determine the range of f.

Answer :

$$f = \left\{ \left(x,\ \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$

$$= \left\{ (0,\ 0),\ \left(\pm 0.5,\ \frac{1}{5} \right),\ \left(\pm 1,\ \frac{1}{2} \right),\ \left(\pm 1.5,\ \frac{9}{13} \right),\ \left(\pm 2,\ \frac{4}{5} \right),\ \left(3,\ \frac{9}{10} \right),\ \left(4,\ \frac{16}{17} \right),\ \ldots \right\}$$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of f = [0, 1)

Question 7:

Let f, g: **R** → **R** be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$.

Answer :

f, g: **R** → **R** is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$

$\therefore (f + g)(x) = 3x - 2$

$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f - g)(x) = -x + 4$

$$\left(\frac{f}{g} \right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0,\ x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g} \right)(x) = \frac{x+1}{2x-3},\ 2x - 3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g} \right)(x) = \frac{x+1}{2x-3},\ x \neq \frac{3}{2}$$

Question 8:

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from **Z** to **Z** defined by $f(x) = ax + b$, for some integers a, b. Determine a, b.

Answer :

$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$

$f(x) = ax + b$

$(1, 1) \in f$

$\Rightarrow f(1) = 1$

$\Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$

$(0, -1) \in f$

$\Rightarrow f(0) = -1$

$\Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting $b = -1$ in $a + b = 1$, we obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and -1.

Question 9:

Let R be a relation from **N** to **N** defined by R = $\{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in$ R, for all $a \in \mathbf{N}$ (ii) $(a, b) \in$ R, implies $(b, a) \in$ R

(iii) $(a, b) \in$ R, $(b, c) \in$ R implies $(a, c) \in$ R.

Justify your answer in each case.

Answer :

R = {(a, b): a, $b \in$ **N** and $a = b^2$}

(i) It can be seen that 2 ∈ **N**;however, $2 \neq 2^2 = 4$.

Therefore, the statement "(a, a) ∈ R, for all $a \in$ **N**" is not true.

(ii) It can be seen that (9, 3) ∈ **N** because 9, 3 ∈ **N** and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, (3, 9) ∉ **N**

Therefore, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.

(iii) It can be seen that (9, 3) ∈ R, (16, 4) ∈ R because 9, 3, 16, 4 ∈ **N** and $9 = 3^2$ and $16 = 4^2$.

Now, $9 \neq 4^2 = 16$; therefore, (9, 4) ∉ **N**

Therefore, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

Question 10:

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

Answer :

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

∴A × B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

It is given that f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A × B.

It is observed that f is a subset of A × B.

Thus, f is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation f is not a function.

Question 11:

Let f be the subset of $\mathbf{Z} \times \mathbf{Z}$ defined by $f = \{(ab, a + b): a, b \in \mathbf{Z}\}$. Is f a function from **Z** to **Z**: justify your answer.

Answer :

The relation f is defined as $f = \{(ab, a + b): a, b \in \mathbf{Z}\}$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since $2, 6, -2, -6 \in \mathbf{Z}$, $(2 \times 6, 2 + 6), (-2 \times -6, -2 + (-6)) \in f$

i.e., $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and –8. Thus, relation f is not a function.

Question 12:

Let A = {9, 10, 11, 12, 13} and let f: A → **N** be defined by $f(n)$ = the highest prime factor of n. Find the range of f.

Answer :

A = {9, 10, 11, 12, 13}

f: A → **N** is defined as

$f(n)$ = The highest prime factor of n

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

$\therefore f(9)$ = The highest prime factor of 9 = 3

$f(10)$ = The highest prime factor of 10 = 5

$f(11)$ = The highest prime factor of 11 = 11

$f(12)$ = The highest prime factor of 12 = 3

$f(13)$ = The highest prime factor of 13 = 13

The range of f is the set of all $f(n)$, where $n \in A$.

$\therefore$Range of $f = \{3, 5, 11, 13\}$