Chapter-6 Linear Inequalities

Miscellaneous

Question 1:

Solve the inequality $2 \leq 3x - 4 \leq 5$

Answer :

$2 \leq 3x - 4 \leq 5$

$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$

$\Rightarrow 6 \leq 3x \leq 9$

$\Rightarrow 2 \leq x \leq 3$

Thus, all the real numbers, x, which are greater than or equal to 2 but less than or equal to 3, are the solutions of the given inequality. The solution set for the given inequalityis [2, 3].

Question 2:

Solve the inequality $6 \leq -3(2x - 4) < 12$

Answer :

$6 \leq -3(2x - 4) < 12$

$\Rightarrow 2 \leq -(2x - 4) < 4$

$\Rightarrow -2 \geq 2x - 4 > -4$

$\Rightarrow 4 - 2 \geq 2x > 4 - 4$

$\Rightarrow 2 \geq 2x > 0$

$\Rightarrow 1 \geq x > 0$

Thus, the solution set for the given inequalityis (0, 1].

Question 3:

Solve the inequality $-3 \le 4 - \frac{7x}{2} \le 18$

Answer :

$$-3 \le 4 - \frac{7x}{2} \le 18$$

$$\Rightarrow -3 - 4 \le -\frac{7x}{2} \le 18 - 4$$

$$\Rightarrow -7 \le -\frac{7x}{2} \le 14$$

$$\Rightarrow 7 \ge \frac{7x}{2} \ge -14$$

$$\Rightarrow 1 \ge \frac{x}{2} \ge -2$$

$$\Rightarrow 2 \ge x \ge -4$$

Thus, the solution set for the given inequalityis [–4, 2].

Question 4:

Solve the inequality $-15 < \frac{3(x-2)}{5} \le 0$

Answer :

$$-15 < \frac{3(x-2)}{5} \le 0$$

$\Rightarrow -75 < 3(x - 2) \le 0$

$\Rightarrow -25 < x - 2 \le 0$

$\Rightarrow -25 + 2 < x \le 2$

$\Rightarrow -23 < x \le 2$

Thus, the solution set for the given inequalityis (–23, 2].

Question 5:

Solve the inequality $-12 < 4 - \frac{3x}{-5} \le 2$

Answer :

$$-12 < 4 - \frac{3x}{-5} \le 2$$
$$\Rightarrow -12 - 4 < \frac{-3x}{-5} \le 2 - 4$$
$$\Rightarrow -16 < \frac{3x}{5} \le -2$$
$$\Rightarrow -80 < 3x \le -10$$
$$\Rightarrow \frac{-80}{3} < x \le \frac{-10}{3}$$

Thus, the solution set for the given inequalityis $\left(\frac{-80}{3}, \frac{-10}{3}\right]$.

Question 6:

Solve the inequality $7 \le \frac{(3x+11)}{2} \le 11$

Answer :

$$7 \le \frac{(3x+11)}{2} \le 11$$
$$\Rightarrow 14 \le 3x + 11 \le 22$$
$$\Rightarrow 14 - 11 \le 3x \le 22 - 11$$
$$\Rightarrow 3 \le 3x \le 11$$
$$\Rightarrow 1 \le x \le \frac{11}{3}$$

Thus, the solution set for the given inequalityis $\left[1, \frac{11}{3}\right]$.

Question 7:

Solve the inequalities and represent the solution graphically on number line: $5x + 1 > -24$, $5x - 1 < 24$

Answer :

$5x + 1 > -24$

$\Rightarrow 5x > -25$

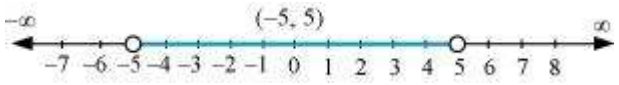
$\Rightarrow x > -5 \ldots (1)$

$5x - 1 < 24$

$\Rightarrow 5x < 25$

$\Rightarrow x < 5 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is (–5, 5). The solution of the given system of inequalities can be represented on number line as



Question 8:

Solve the inequalities and represent the solution graphically on number line: $2(x - 1) < x + 5$, $3(x + 2) > 2 - x$

Answer :

$2(x - 1) < x + 5$

$\Rightarrow 2x - 2 < x + 5$

$\Rightarrow 2x - x < 5 + 2$

$\Rightarrow x < 7 \ldots (1)$

$3(x + 2) > 2 - x$

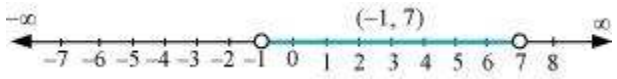
$\Rightarrow 3x + 6 > 2 - x$

$\Rightarrow 3x + x > 2 - 6$

$\Rightarrow 4x > -4$

$\Rightarrow x > -1 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is (–1, 7). The solution of the given system of inequalities can be represented on number line as



Question 9:

Solve the following inequalities and represent the solution graphically on number line:

$3x - 7 > 2(x - 6), 6 - x > 11 - 2x$

Answer :

$3x - 7 > 2(x - 6)$

$\Rightarrow 3x - 7 > 2x - 12$

$\Rightarrow 3x - 2x > -12 + 7$

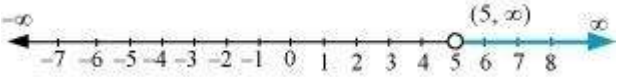
$\Rightarrow x > -5 \ldots (1)$

$6 - x > 11 - 2x$

$\Rightarrow -x + 2x > 11 - 6$

$\Rightarrow x > 5 \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(5, \infty)$. The solution of the given system of inequalities can be represented on number line as



Question 10:

Solve the inequalities and represent the solution graphically on number line: $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x + 47$

Answer :

$5(2x - 7) - 3(2x + 3) \leq 0$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$

$\Rightarrow 4x - 44 \leq 0$

$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11 \ldots (1)$

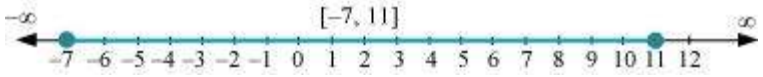
$2x + 19 \leq 6x + 47$

$\Rightarrow 19 - 47 \leq 6x - 2x$

$\Rightarrow -28 \leq 4x$

$\Rightarrow -7 \leq x \ldots (2)$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $[-7, 11]$. The solution of the given system of inequalities can be represented on number line as



Question 11:

A solution is to be kept between 68°F and 77°F. What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{8}C + 32$?

Answer :

Since the solution is to be kept between 68°F and 77°F,

$68 < F < 77$

Putting $F = \frac{9}{5}C + 32,$ we obtain

$$68 < \frac{9}{5}C + 32 < 77$$
$$\Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$
$$\Rightarrow 36 < \frac{9}{5}C < 45$$
$$\Rightarrow 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$
$$\Rightarrow 20 < C < 25$$

Thus, the required range of temperature in degree Celsius is between 20°C and 25°C.

Question 12:

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Answer :

Let x litres of 2% boric acid solution is required to be added.

Then, total mixture = $(x + 640)$ litres

This resulting mixture is to be more than 4% but less than 6% boric acid.

$\therefore$2%x + 8% of 640 > 4% of $(x + 640)$

And, 2% x + 8% of 640 < 6% of $(x + 640)$

2%x + 8% of 640 > 4% of $(x + 640)$

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) > \frac{4}{100}(x + 640)$$

$\Rightarrow 2x + 5120 > 4x + 2560$

$\Rightarrow 5120 - 2560 > 4x - 2x$

$\Rightarrow 5120 - 2560 > 2x$

$\Rightarrow 2560 > 2x$

$\Rightarrow 1280 > x$

2% x + 8% of 640 < 6% of (x + 640)

$$\frac{2}{100}x+\frac{8}{100}(640)<\frac{6}{100}(x+640)$$

$\Rightarrow 2x + 5120 < 6x + 3840$

$\Rightarrow 5120 - 3840 < 6x - 2x$

$\Rightarrow 1280 < 4x$

$\Rightarrow 320 < x$

$\therefore 320 < x < 1280$

Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

Question 13:

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Answer :

Let x litres of water is required to be added.

Then, total mixture = (x + 1125) litres

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres.

This resulting mixture will contain more than 25% but less than 30% acid content.

$\therefore$30% of (1125 + x) > 45% of 1125

And, 25% of (1125 + x) < 45% of 1125

30% of (1125 + x) > 45% of 1125

$$\Rightarrow \frac{30}{100}(1125+x) > \frac{45}{100}\times 1125$$
$$\Rightarrow 30(1125+x) > 45\times 1125$$
$$\Rightarrow 30\times 1125 + 30x > 45\times 1125$$
$$\Rightarrow 30x > 45\times 1125 - 30\times 1125$$
$$\Rightarrow 30x > (45-30)\times 1125$$
$$\Rightarrow x > \frac{15\times 1125}{30} = 562.5$$

25% of $(1125 + x) < 45\%$ of 1125

$$\Rightarrow \frac{25}{100}(1125+x) < \frac{45}{100}\times 1125$$
$$\Rightarrow 25(1125+x) > 45\times 1125$$
$$\Rightarrow 25\times 1125 + 25x > 45\times 1125$$
$$\Rightarrow 25x > 45\times 1125 - 25\times 1125$$
$$\Rightarrow 25x > (45-25)\times 1125$$
$$\Rightarrow x > \frac{20\times 1125}{25} = 900$$

$\therefore 562.5 < x < 900$

Thus, the required number of litres of water that is to be added will have to be more than 562.5 but less than 900.

Question 14:

IQ of a person is given by the formula

$$IQ = \frac{MA}{CA}\times 100,$$

Where MA is mental age and CA is chronological age. If $80 \le IQ \le 140$ for a group of 12 years old children, find the range of their mental age.

Answer :

It is given that for a group of 12 years old children, $80 \le IQ \le 140$ … (i)

For a group of 12 years old children, CA = 12 years

$$IQ = \frac{MA}{12} \times 100$$

Putting this value of IQ in (i), we obtain

$$80 \le \frac{MA}{12} \times 100 \le 140$$
$$\Rightarrow 80 \times \frac{12}{100} \le MA \le 140 \times \frac{12}{100}$$
$$\Rightarrow 9.6 \le MA \le 16.8$$

Thus, the range of mental age of the group of 12 years old children is $9.6 \le MA \le 16.8$.