Question 1:

Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, −1), (4, 3, −1).

Answer :

Let OA be the line joining the origin, O (0, 0, 0), and the point, A (2, 1, 1).

Also, let BC be the line joining the points, B (3, 5, −1) and C (4, 3, −1).

The direction ratios of OA are 2, 1, and 1 and of BC are (4 − 3) = 1, (3 − 5) = −2, and (−1 + 1) = 0

OA is perpendicular to BC, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1\,(-2) + 1 \times 0 = 2 - 2 = 0$

Thus, OA is perpendicular to BC.

Question 2:

If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

Answer :

It is given that l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines. Therefore,

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \qquad ...(1)$$

$$l_1^2 + m_1^2 + n_1^2 = 1 \qquad ...(2)$$

$$l_2^2 + m_2^2 + n_2^2 = 1 \qquad ...(3)$$

Let l, m, n be the direction cosines of the line which is perpendicular to the line with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2.

$$\therefore ll_1 + mm_1 + nn_1 = 0$$

$$ll_2 + mm_2 + nn_2 = 0$$

$$\therefore \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_l}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_l)^2}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_2)^2}$$

$$= \frac{l^2 + m^2 + n^2}{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_l)^2} \quad ...(4)$$

l, *m*, *n* are the direction cosines of the line.

$\therefore l^2 + m^2 + n^2 = 1$ … (5)

It is known that,

$$(l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1l_2 + m_1m_2 + n_1n_2)^2$$

$$= (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$$

From (1), (2), and (3), we obtain

$$\Rightarrow 1.1 - 0 = (m_1n_2 + m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$$

$$\therefore (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2 = 1 \quad ...(6)$$

Substituting the values from equations (5) and (6) in equation (4), we obtain

$$\frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_2l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_1)^2} = 1$$

$$\Rightarrow l = m_1n_2 - m_2n_1, m = n_1l_2 - n_2l_1, n = l_1m_2 - l_2m_1$$

Thus, the direction cosines of the required line are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1, \text{and}\ l_1m_2 - l_2m_1.$

Question 3:

Find the angle between the lines whose direction ratios are *a*, *b*, *c* and *b* − *c*,

$c - a$, $a - b$.

Answer :

The angle Q between the lines with direction cosines, a, b, c and $b - c$, $c - a$,

$a - b$, is given by,

$$\cos Q = \left| \frac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}+\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}} \right|$$
$$\Rightarrow \cos Q = 0$$
$$\Rightarrow Q = \cos^{-1} 0$$
$$\Rightarrow Q = 90^\circ$$

Thus, the angle between the lines is 90°.

Question 4:

Find the equation of a line parallel to x-axis and passing through the origin.

Answer :

The line parallel to x-axis and passing through the origin is x-axis itself.

Let A be a point on x-axis. Therefore, the coordinates of A are given by (a, 0, 0), where $a \in$ R.

Direction ratios of OA are $(a - 0) = a, 0, 0$

The equation of OA is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$
$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

Thus, the equation of line parallel to x-axis and passing through origin is

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

Question 5:

If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (−4, 3, −6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Answer :

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and

(2, 9, 2) respectively.

The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4

The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180°.

Question 6:

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of *k*.

Answer :

The direction of ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are −3, 2*k*, 2 and 3*k*, 1, −5 respectively.

It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$
$$\Rightarrow -9k + 2k - 10 = 0$$
$$\Rightarrow 7k = -10$$
$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for $k=-\frac{10}{7}$, the given lines are perpendicular to each other.

Question 7:

Find the vector equation of the plane passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0$

Answer :

The position vector of the point (1, 2, 3) is $\vec{r_1}=\hat{i}+2\hat{j}+3\hat{k}$

The direction ratios of the normal to the plane, $\vec{r}\cdot\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0$, are 1, 2, and −5 and the normal vector is $\vec{N}=\hat{i}+2\hat{j}-5\hat{k}$

The equation of a line passing through a point and perpendicular to the given plane is given by, $\vec{l}=\vec{r}+\lambda\vec{N},\ \lambda\in R$

$$\Rightarrow\vec{l}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-5\hat{k}\right)$$

Question 8:

Find the equation of the plane passing through (*a*, *b*, *c*) and parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$

Answer :

Any plane parallel to the plane, $\vec{r_1}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$, is of the form

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda \qquad ...(1)$$

The plane passes through the point (*a*, *b*, *c*). Therefore, the position vector $\vec{r}$ of this point is $\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$

Therefore, equation (1) becomes

$$\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda$$
$$\Rightarrow a+b+c=\lambda$$

Substituting $\lambda=a+b+c$ in equation (1), we obtain

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c \qquad ...(2)$$

This is the vector equation of the required plane.

Substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ in equation (2), we obtain

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$$
$$\Rightarrow x+y+z=a+b+c$$

Question 9:

Find the shortest distance between lines $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

and $\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$.

Answer :

The given lines are

$$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right) \qquad ...(1)$$
$$\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right) \qquad ...(2)$$

It is known that the shortest distance between two lines, $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$, is given by

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| \qquad ...(3)$$

Comparing $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ to equations (1) and (2), we obtain

$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$
$\vec{a}_2 = -4\hat{i} - \hat{k}$
$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = \left(-4\hat{i} - \hat{k}\right) - \left(6\hat{i} + 2\hat{j} + 2\hat{k}\right) = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = (4+4)\hat{i} - (-2-6)\hat{j} + (-2+6)\hat{k} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(8)^2 + (8)^2 + (4)^2} = 12$$

$$\left(\vec{b}_1 \times \vec{b}_2\right) \cdot \left(\vec{a}_2 - \vec{a}_1\right) = \left(8\hat{i} + 8\hat{j} + 4\hat{k}\right) \cdot \left(-10\hat{i} - 2\hat{j} - 3\hat{k}\right) = -80 - 16 - 12 = -108$$

Substituting all the values in equation (1), we obtain

$$d = \left|\frac{-108}{12}\right| = 9$$

Therefore, the shortest distance between the two given lines is 9 units

Question 10:

Find the coordinates of the point where the line through (5, 1, 6) and

(3, 4, 1) crosses the YZ-plane

Answer :

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$
$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k\left(\text{say}\right)$$
$$\Rightarrow x=5-2k,\ y=3k+1,\ z=6-5k$$

Any point on the line is of the form $(5 - 2k, 3k + 1, 6 - 5k)$.

The equation of YZ-plane is $x = 0$

Since the line passes through YZ-plane,

$5 - 2k = 0$

$$\Rightarrow k=\frac{5}{2}$$

$$\Rightarrow 3k+1=3\times\frac{5}{2}+1=\frac{17}{2}$$
$$6-5k=6-5\times\frac{5}{2}=\frac{-13}{2}$$

Therefore, the required point is $\left(0,\frac{17}{2},\frac{-13}{2}\right)$.

Question 11:

Find the coordinates of the point where the line through (5, 1, 6) and

(3, 4, 1) crosses the ZX − plane.

Answer :

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$
$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k\left(\text{say}\right)$$
$$\Rightarrow x=5-2k,\ y=3k+1,\ z=6-5k$$

Any point on the line is of the form $(5 - 2k, 3k + 1, 6 - 5k)$.

Since the line passes through ZX-plane,

$$3k+1=0$$
$$\Rightarrow k=-\frac{1}{3}$$

$$\Rightarrow 5-2k=5-2\left(-\frac{1}{3}\right)=\frac{17}{3}$$
$$6-5\text{k}=6-5\left(-\frac{1}{3}\right)=\frac{23}{3}$$

Therefore, the required point is $\left(\frac{17}{3},0,\frac{23}{3}\right)$.

Question 11:

Find the coordinates of the point where the line through (5, 1, 6) and

(3, 4, 1) crosses the ZX − plane.

Answer :

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$
$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k\,(\text{say})$$
$$\Rightarrow x=5-2k,\ y=3k+1,\ z=6-5k$$

Any point on the line is of the form (5 − 2k, 3k + 1, 6 −5k).

Since the line passes through ZX-plane,

$$3k+1=0$$
$$\Rightarrow k=-\frac{1}{3}$$

$$\Rightarrow 5-2k=5-2\left(-\frac{1}{3}\right)=\frac{17}{3}$$
$$6-5\text{k}=6-5\left(-\frac{1}{3}\right)=\frac{23}{3}$$

Therefore, the required point is $\left(\frac{17}{3},0,\frac{23}{3}\right)$.

Question 14:

If the points (1, 1, p) and (−3, 0, 1) be equidistant from the plane $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$, then find the value of p.

Answer :

The position vector through the point (1, 1, p) is $\vec{a}_1=\hat{i}+\hat{j}+p\hat{k}$

Similarly, the position vector through the point (−3, 0, 1) is

$$\vec{a}_2=-4\hat{i}+\hat{k}$$

The equation of the given plane is $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$

It is known that the perpendicular distance between a point whose position vector is $\vec{a}$ and the plane, $\vec{r}\cdot\vec{N}=d$, is given by, $D=\frac{\left|\vec{a}\cdot\vec{N}-d\right|}{\left|\vec{N}\right|}$

Here, $\vec{N}=3\hat{i}+4\hat{j}-12\hat{k}$ and $d=-13$

Therefore, the distance between the point (1, 1, p) and the given plane is

$$D_1=\frac{\left|\left(\hat{i}+\hat{j}+p\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$
$$\Rightarrow D_1=\frac{\left|3+4-12p+13\right|}{\sqrt{3^2+4^2+\left(-12\right)^2}}$$
$$\Rightarrow D_1=\frac{\left|20-12p\right|}{13} \qquad ...(1)$$

Similarly, the distance between the point (−3, 0, 1) and the given plane is

$$D_2=\frac{\left|\left(-3\hat{i}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$
$$\Rightarrow D_2=\frac{\left|-9-12+13\right|}{\sqrt{3^2+4^2+\left(-12\right)^2}}$$
$$\Rightarrow D_2=\frac{8}{13} \qquad ...(2)$$

It is given that the distance between the required plane and the points, (1, 1, p) and (−3, 0, 1), is equal.

$\therefore D_1 = D_2$

$$\Rightarrow \frac{\left|20-12p\right|}{13}=\frac{8}{13}$$

$$\Rightarrow 20-12p=8 \text{ or } -\left(20-12p\right)=8$$
$$\Rightarrow 12p=12 \text{ or } 12p=28$$
$$\Rightarrow p=1 \text{ or } p=\frac{7}{3}$$

Question 15:

Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to *x*-axis.

Answer :

The given planes are

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$$
$$\Rightarrow \vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1=0$$
$$\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$$

The equation of any plane passing through the line of intersection of these planes is

$$\left[\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4\right]=0$$
$$\vec{r}\cdot\left[(2\lambda+1)\hat{i}+(3\lambda+1)\hat{j}+(1-\lambda)\hat{k}\right]+(4\lambda+1)=0 \qquad ...(1)$$

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$, and $(1 - \lambda)$.

The required plane is parallel to *x*-axis. Therefore, its normal is perpendicular to *x*-axis.

The direction ratios of *x*-axis are 1, 0, and 0.

$$\therefore 1.(2\lambda+1)+0(3\lambda+1)+0(1-\lambda)=0$$
$$\Rightarrow 2\lambda+1=0$$
$$\Rightarrow \lambda=-\frac{1}{2}$$

Substituting $\lambda=-\frac{1}{2}$ in equation (1), we obtain

$$\Rightarrow \vec{r}\cdot\left[-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]+(-3)=0$$
$$\Rightarrow \vec{r}\left(\hat{j}-3\hat{k}\right)+6=0$$

Therefore, its Cartesian equation is $y - 3z + 6 = 0$

This is the equation of the required plane.

Question 16:

If O be the origin and the coordinates of P be (1, 2, −3), then find the equation of the plane passing through P and perpendicular to OP.

Answer :

The coordinates of the points, O and P, are (0, 0, 0) and (1, 2, −3) respectively.

Therefore, the direction ratios of OP are (1 − 0) = 1, (2 − 0) = 2, and (−3 − 0) = −3

It is known that the equation of the plane passing through the point (x_1, y_1 z_1) is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ where, a, b, and c are the direction ratios of normal.

Here, the direction ratios of normal are 1, 2, and −3 and the point P is (1, 2, −3).

Thus, the equation of the required plane is

$$1(x-1)+2(y-2)-3(z+3)=0$$
$$\Rightarrow x+2y-3z-14=0$$

Question 17:

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Answer :

The equations of the given planes are

$$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0 \qquad ...(1)$$
$$\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0 \qquad ...(2)$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$\left[\vec{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right)+5\right]=0$$

$$\vec{r}\cdot\left[(2\lambda+1)\hat{i}+(\lambda+2)\hat{j}+(3-\lambda)\hat{k}\right]+(5\lambda-4)=0 \qquad ...(3)$$

The plane in equation (3) is perpendicular to the plane, $\vec{r}\cdot\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

$$\therefore 5(2\lambda+1)+3(\lambda+2)-6(3-\lambda)=0$$
$$\Rightarrow 19\lambda-7=0$$
$$\Rightarrow \lambda=\frac{7}{19}$$

Substituting $\lambda=\frac{7}{19}$ in equation (3), we obtain

$$\Rightarrow \vec{r}\cdot\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]\frac{-41}{19}=0$$

$$\Rightarrow \vec{r}\cdot\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0 \qquad ...(4)$$

This is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ in equation (3).

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$$
$$\Rightarrow 33x+45y+50z-41=0$$

Question 18:

Find the distance of the point (−1, −5, −10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Answer :

The equation of the given line is

$$\vec{r}\cdot = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right) \quad ...(1)$$

The equation of the given plane is

$$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5 \quad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$
$$\Rightarrow \left[(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$
$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$
$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is
$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance *d* between the points, (2, −1, 2) and (−1, −5, −10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

Question 19:

Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes
$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}\cdot\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$.

Answer :

Let the required line be parallel to vector $\vec{b}$ given by,

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of the point (1, 2, 3) is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{r}\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \qquad ...(1)$$

The equations of the given planes are

$$\vec{r}\cdot\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5 \qquad ...(2)$$

$$\vec{r}\cdot\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6 \qquad ...(3)$$

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$$\Rightarrow \left(\hat{i} - \hat{j} + 2\hat{k}\right)\cdot\lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$$

$$\Rightarrow \lambda\left(b_1 - b_2 + 2b_3\right) = 0$$

$$\Rightarrow b_1 - b_2 + 2b_3 = 0 \qquad ...(4)$$

Similarly, $\left(3\hat{i} + \hat{j} + \hat{k}\right)\cdot\lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$

$$\Rightarrow \lambda\left(3b_1 + b_2 + b_3\right) = 0$$

$$\Rightarrow 3b_1 + b_2 + b_3 = 0 \qquad ...(5)$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-1)\times1 - 1\times2} = \frac{b_2}{2\times3 - 1\times1} = \frac{b_3}{1\times1 - 3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of $\vec{b}$ are −3, 5, and 4.

$$\therefore \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of $\vec{b}$ in equation (1), we obtain

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

This is the equation of the required line.

Question 20:

Find the vector equation of the line passing through the point (1, 2, − 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Answer :

Let the required line be parallel to the vector $\vec{b}$ given by, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of the point (1, 2, − 4) is $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

The equation of the line passing through (1, 2, −4) and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\Rightarrow \vec{r}\left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \quad ...(1)$$

The equations of the lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad ...(2)$$
$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad ...(3)$$

Line (1) and line (2) are perpendicular to each other.

$$\therefore 3b_1 - 16b_2 + 7b_3 = 0 \quad ...(4)$$

Also, line (1) and line (3) are perpendicular to each other.

$$\therefore 3b_1 + 8b_2 - 5b_3 = 0 \quad ...(5)$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-16)(-5)-8\times 7}=\frac{b_2}{7\times 3-3(-5)}=\frac{b_3}{3\times 8-3(-16)}$$
$$\Rightarrow \frac{b_1}{24}=\frac{b_2}{36}=\frac{b_3}{72}$$
$$\Rightarrow \frac{b_1}{2}=\frac{b_2}{3}=\frac{b_3}{6}$$

∴Direction ratios of $\vec{b}$ are 2, 3, and 6.

$$\therefore \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$$

Substituting $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ in equation (1), we obtain

$$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

This is the equation of the required line.

Question 21:

Prove that if a plane has the intercepts *a*, *b*, *c* and is at a distance of *P* units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

Answer :

The equation of a plane having intercepts *a*, *b*, *c* with *x*, *y*, and *z* axes respectively is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ...(1)$$

The distance (*p*) of the plane from the origin is given by,

$$p=\left|\frac{\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}\right|$$

$$\Rightarrow p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

Question 22:

Distance between the two planes: $2x+3y+4z=4$ and $4x+6y+8z=12$ is

(A)2 units (B)4 units (C)8 units

(D) $\frac{2}{\sqrt{29}}$ units

Answer :

The equations of the planes are

$$2x+3y+4z=4 \qquad ...(1)$$

$$4x+6y+8z=12$$

$$\Rightarrow 2x+3y+4z=6 \qquad ...(2)$$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes, $ax + by + cz = d_1$ and $ax + by + cz = d_2$, is given by,

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\Rightarrow D = \left| \frac{6 - 4}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right|$$

$$D = \frac{2}{\sqrt{29}}$$

Thus, the distance between the lines is $\frac{2}{\sqrt{29}}$ units.

Hence, the correct answer is D.

Question 23:

The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular (B) Parallel (C) intersect y-axis

(C) passes through $\left(0, 0, \frac{5}{4}\right)$

Answer :

The equations of the planes are

$2x - y + 4z = 5$ … (1)

$5x - 2.5y + 10z = 6$ … (2)

It can be seen that,

$$\frac{a_1}{a_2} = \frac{2}{5}$$

$$\frac{b_1}{b_2} = \frac{-1}{-2.5} = \frac{2}{5}$$

$$\frac{c_1}{c_2} = \frac{4}{10} = \frac{2}{5}$$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the given planes are parallel.

Hence, the correct answer is B.